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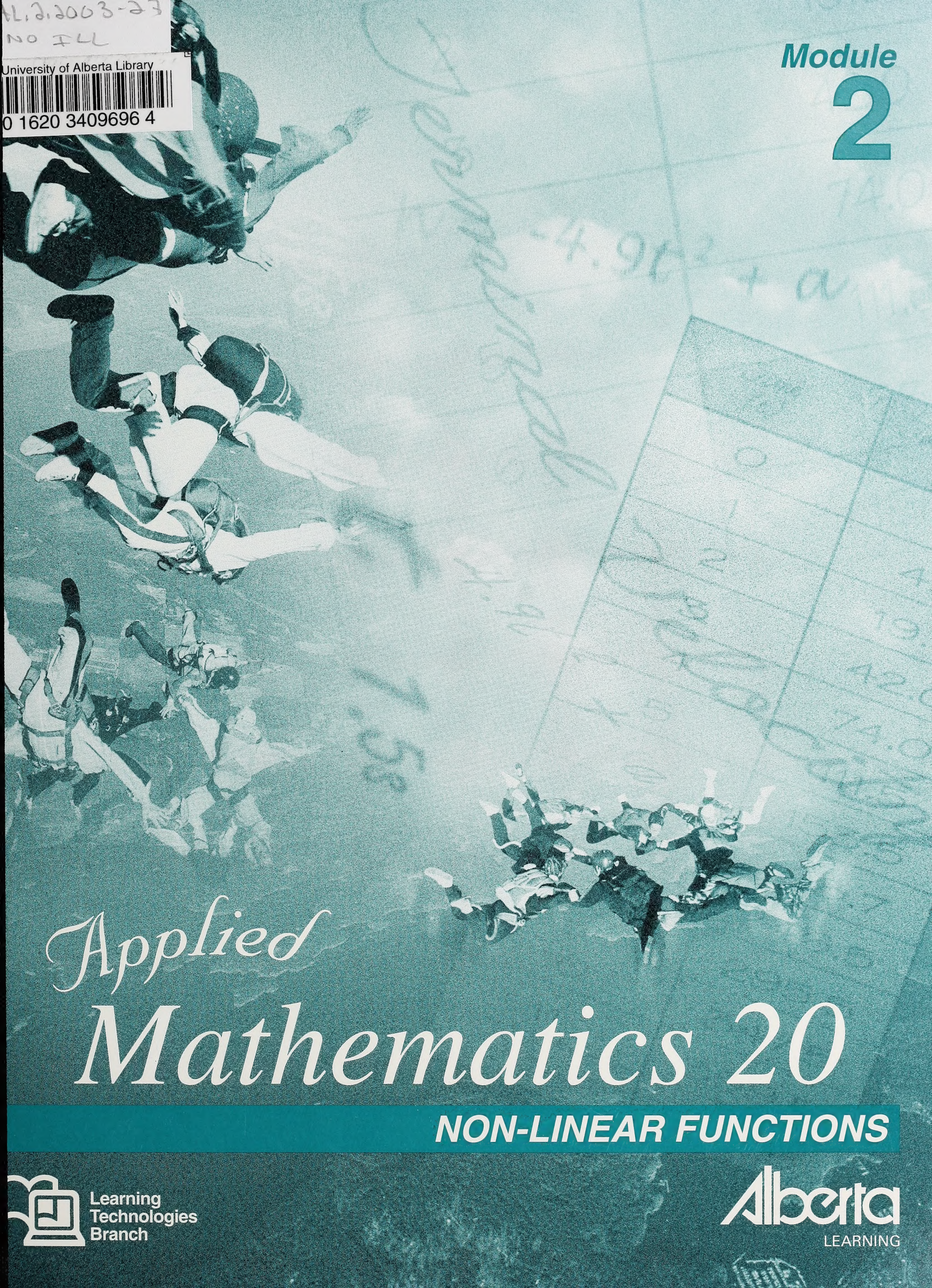
Module 2



Applied Mathematics 20

NON-LINEAR FUNCTIONS

Learning Technologies Branch

Alberta
LEARNING

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Module
2

Applied Mathematics 20

NON-LINEAR FUNCTIONS

Learning Technologies Branch

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Applied Mathematics 20
Module 2: Non-Linear Functions
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-1905-5

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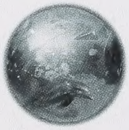
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	



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- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

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Welcome

Applied Mathematics 20

Module 1
Graphs

Module 2
Non-Linear Functions

Module 3
Linear Systems

Module 4
Linear Programming

Module 5
Finance

Module 6
Circles

Module 7
Measurement

Welcome to Module 2. We hope you'll enjoy your study of Non-Linear Functions.

Applied Mathematics 20 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

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Submit the Module Assignment.

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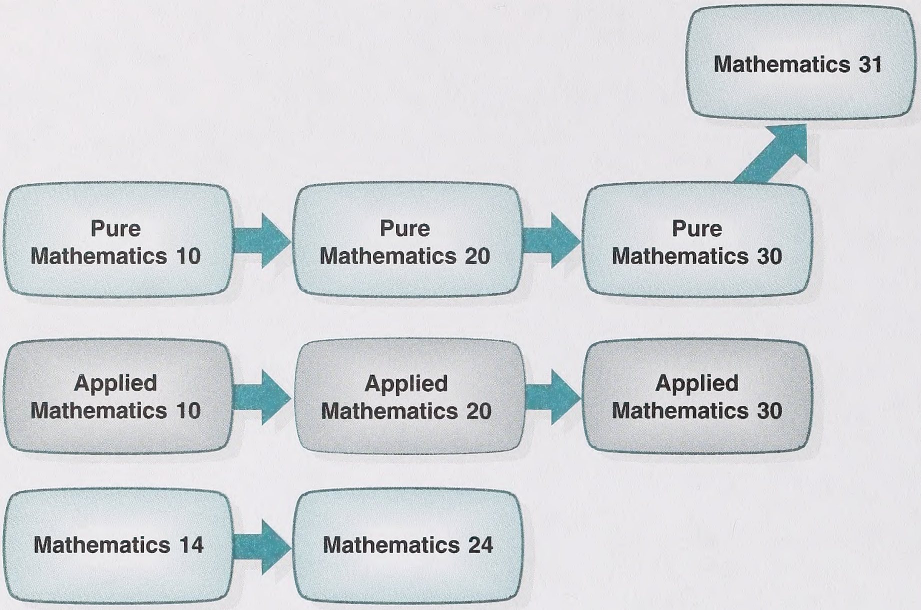
Introduction to Applied Mathematics 20

Applied Mathematics 20 is the second course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different career plans. For example, Pure Mathematics 30 is a prerequisite for admission to many university programs. Many colleges and technical institutes, however, will admit students who have successfully completed Applied Mathematics 30.

You may find it helpful to read "Questions and Answers About Senior High School Mathematics" and "The New Senior High School Mathematics Program and Post-Secondary Studies." These documents can be found at the following Internet site:

http://www.learning.gov.ab.ca/k_12/curriculum/bySubject/math

Before enrolling in Applied Mathematics 20, it is recommended that you talk with a school counsellor about your career plans.

TRANSFERRING FROM THE APPLIED PROGRAM

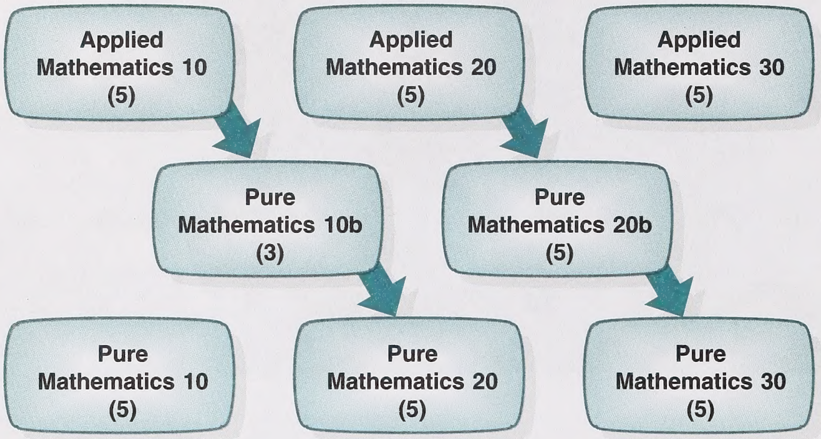
You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

The following table shows some common and independent topics.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics.

If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may have to take a three-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20, you may have to take a five-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.

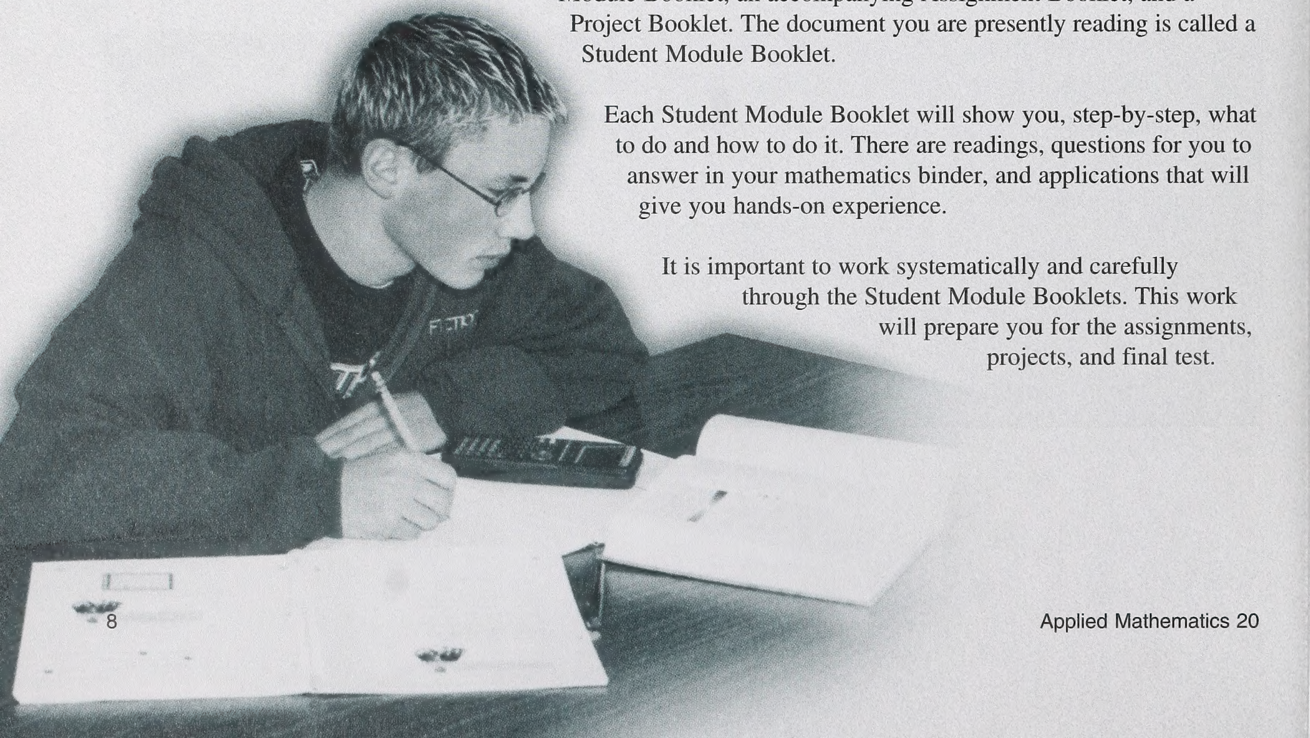


STRATEGIES FOR COMPLETING APPLIED MATHEMATICS 20

For each module in Applied Mathematics 20, there is a Student Module Booklet, an accompanying Assignment Booklet, and a Project Booklet. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step-by-step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the assignments, projects, and final test.



Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also store your marked assignments here.

- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.

- Keep a section of your binder to record new skills and concepts, as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words.

  Record useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.

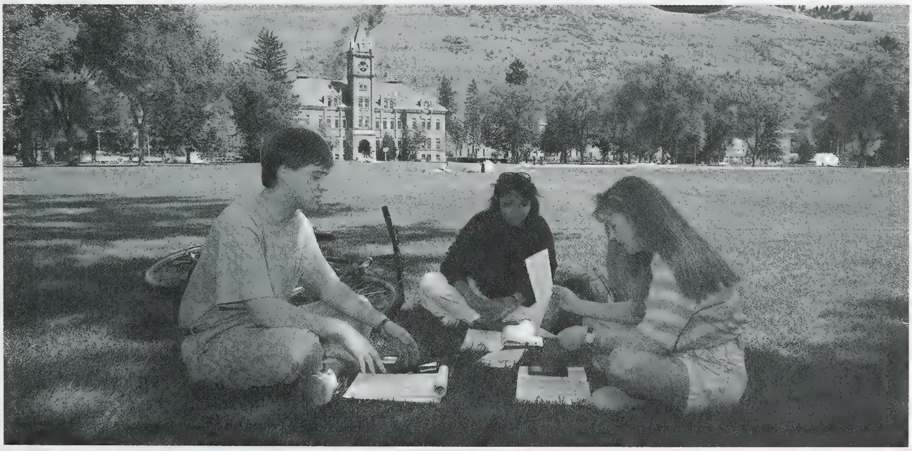
- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an "aha!" experience), or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 20 to be your study partner. You will find that having a friend to discuss mathematical ideas with will make your studying more enjoyable.



Resources You Will Need

In addition to the distance learning materials for Applied Mathematics 20, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 11 Source Book* Western Canadian Edition, published by Addison Wesley Longman Ltd. (2000)
- a binder, lined loose-leaf paper, graph paper, dividers, pencils, eraser
- metric and imperial measuring devices, such as a ruler, yardstick, metre-stick, and tape measure
- a mathematical instrument set (compass, protractor, and triangles)
- a computer with a spreadsheet program

  Note: Two popular spreadsheet programs are *ClarisWorks*™ and Microsoft® *Excel*.

- a graphing calculator

  Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Texas Instruments	Sharp	Casio	Hewlett-Packard
TI-82**	EL-9600C	Algebra FX 2.0	HP 39g
TI-83	EL-9600*	CFX-9850 GA-Plus*	
TI-83 Plus	EL-9200*	CFX-9850 G*	
TI-86	EL-9300*	CFX-9800 G*	
TI-89		FX-9700 series*	
TI-92*			
TI-92 Plus			

*no longer commercially available

**The TI-82 calculator will remain on the approved list for the 2000–2001 and 2001–2002 school years and will then be deleted from the approved list.

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you obtain the video program *The TI-83 Graphing Calculator Video Tutor*.

Many of the resources that you will need may be purchased locally or from the Learning Resources Centre (LRC). Following is the LRC website:

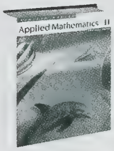
http://www.lrc.learning.gov.ab.ca

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

Visual Cues

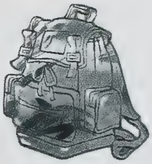
You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the following explanations to discover what the various icons prompt you to do.

- Refer to the textbook.

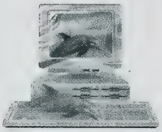


- Refer to the Applied Mathematics 20 CD.

- Use mathematical instruments or measuring tools.

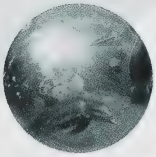


- Work with a computer.



- Complete the module project or assignment.

- Explore the Internet.



Remember: Any Internet website address given in this module is subject to change.

MODULE OVERVIEW

Canmore: Readying for the Future

The town of Canmore used to be a place where virtually all the faces at the corner store were familiar.

Not anymore.

Downtown traffic jams and steady streams of pedestrians in the town's commercial districts are signs Canmore is no longer simply the quaint little town nestled beneath the majestic Rocky Mountains.

Those who were around in 1979—when the coal industry collapsed and Canmore Mines Ltd. closed down—could never have envisioned the phenomenal growth that has hit Canmore.

The town's population has more than doubled in the last decade, leaping from 4,419 in 1988 to about 9,500 today. More than 50 per cent of the town's residents have lived there less than five years.

The town's 1997 census showed a 7.4-per-cent jump in population, slightly lower than the five-year average of 7.8 per cent. . . .[1]

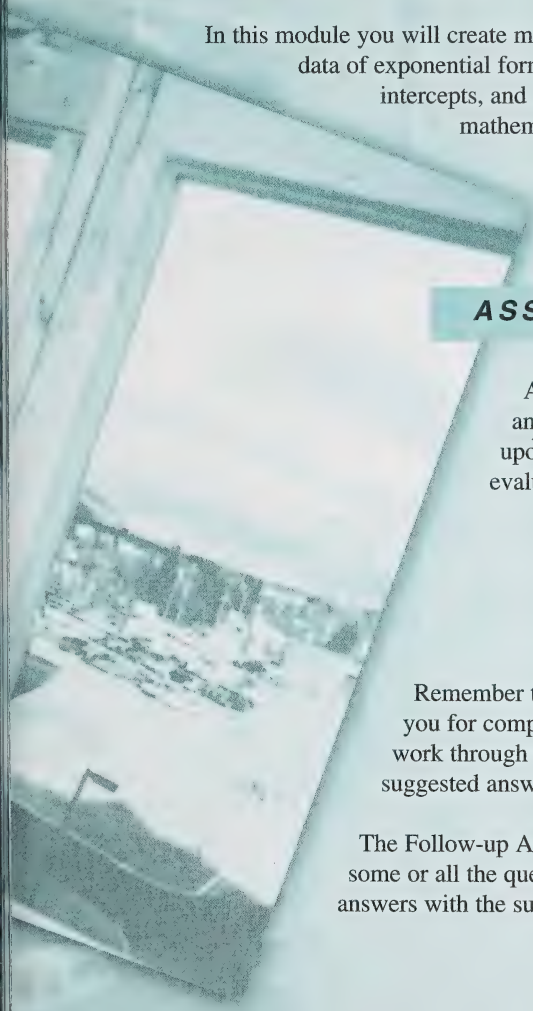
---

[1] "Canmore: Readying for the Future," *The Calgary Herald*, 8 June 1998. Reprinted by permission.

Have you attended a family reunion lately? You probably noticed that the number of people attending was more than it was even a year or two ago. Not only are individual families growing in size, human populations in general are increasing. One location in Western Canada whose population has recently increased dramatically is Canmore, Alberta. From 1991 to 1996 the population of this Rocky Mountain town increased by 47.1%. At this rate, Canmore's population will more than double every decade!

Earth in its entirety is a closed system with respect to population growth—meaning all the materials needed to support the population must come from the planet itself. Apart from solar energy, all energy resources humans use must come from Earth. Cities, on the other hand, are not a closed system with respect to population. As long as there are resources available outside the city limits to sustain an increasing population, cities like Canmore can continue to grow at a breakneck pace.

Although Earth's population has been increasing exponentially, the recognition that Earth is a closed system has led thinkers to wonder what will happen when the maximum population that can be sustained by Earth's resources is reached. In order to predict when Earth's population will reach this maximum, a mathematical model may be used, thus providing insight into population growth.



In this module you will create mathematical models using regression functions; plot and describe data of exponential form; and determine the axis of symmetry, domain and range, intercepts, and vertex of graphs of quadratic functions. Equipped with these mathematical tools, you will be able to identify patterns in data from a variety of contexts and gain a better understanding of Earth's population explosion.

ASSESSMENT

Accompanying this Student Module Booklet is a Project Booklet and an Assignment Booklet. Your grading in this module will be based upon the module project and the module assignment you submit for evaluation. The mark distribution is as follows:

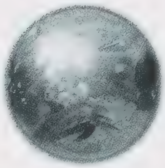
Module Project	**35 marks**
Module Assignment	**65 marks**
TOTAL	**100 marks**

Remember that Activities 1 to 4 in this Student Module Booklet will prepare you for completing the module project and the module assignment. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Follow-up Activities provide extra help and enrichment. You may choose to do some or all the questions in the Follow-up Activities. Again, you should compare your answers with the suggested answers provided in the Appendix.

MODULE PROJECT
THE WORLD'S POPULATION

Beginning the Project



The human population is continually increasing. To see the rate at which the population of Earth is increasing, visit the following website. This website predicts the world's population at that exact second.

http://www.census.gov/cgi-bin/ipc/popclockw

Once there, note the world population; then refresh the page a number of times. Note how the population continually increases.

Since the world's food sources are limited, it seems likely that the number of people on Earth will eventually exceed the number that can be supported by the food available. In 1798, an economist by the name of Thomas Malthus expressed this concern about the population trend. This was a remarkable prediction when the world had a population of just 900 million. Who would have thought at that time that the world's population would eventually exceed 6 billion and still be increasing as it does today? Has Malthus been proven wrong?

Your project for Module 2: Non-Linear Functions is The World's Population. In this project you will report on the historical growth of the world's population based on your own research. You will also fit different mathematical models to the population data. You will then make predictions about future population growth, taking into consideration factors that influence population growth.

In preparing for this project, turn to the Project Booklet and preview what you are expected to submit.

Turn to page 62 of the textbook and read "The World's Population." Once you have read the page, complete the questions posed and begin your research. Store your responses and relevant notes from your research in the project section of your mathematical binder.

Use the Internet, magazines, newspapers, encyclopedias, and other books to research world population. You may find it useful to begin your research by visiting the Internet site of Addison Wesley Longman Ltd. This site is described on page 63 of the textbook.

As you work through the activities, continue your research for this project. You may wish to view the following site. It contains an essay on Malthus's prediction about population growth.

http://www.time.com/time/reports/v21/health/malthus_mag2.html

You may also search for sites regarding Thomas Malthus and population projections at the following site. Enter the term *population* into the website's search engine.

http://www.studyweb.com

You may also try entering *Malthus* into the one of the Internet's search engines.

Thoughout this project, focus on the numbers associated with population growth. In order to describe and analyse these numbers, you will need a good understanding of non-linear functions.

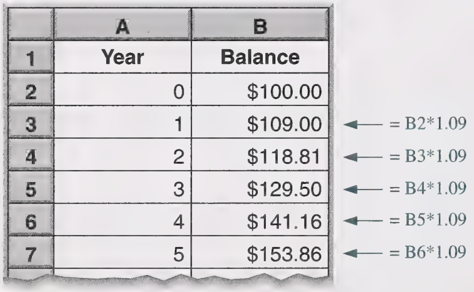
You will be given more direction on how to complete this project later in this module. In the meantime, you may want to talk with other students taking Applied Mathematics 20 or with family members about this project. Remember, the work you submit must be your own.

ACTIVITY 1

Exponential Graphs

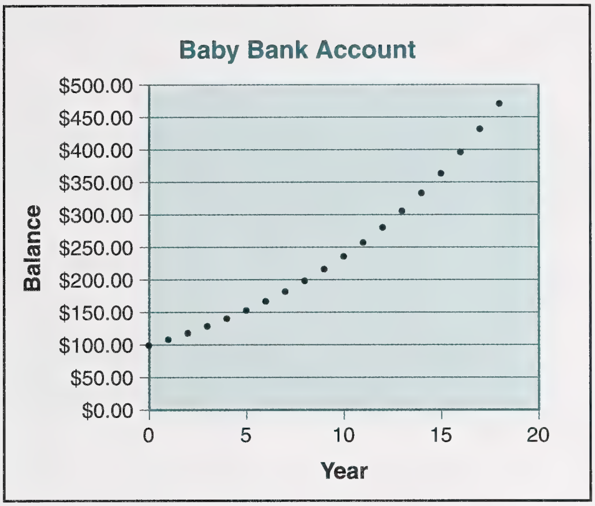
A bank advertises that an account for any baby less than one year old can be opened. This account will pay 9%, compounded annually, for the next 18 years. A deposit of exactly $100 is required to open the account and that no further deposits will be allowed.

The following partial spreadsheet shows how the balance will grow for the first five years.

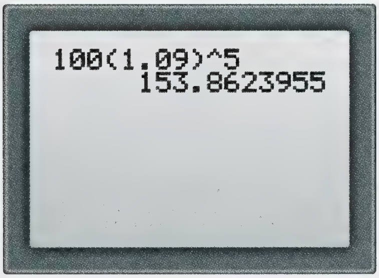


	A	B	
1	Year	Balance	
2	0	$100.00	
3	1	$109.00	← = B2*1.09
4	2	$118.81	← = B3*1.09
5	3	$129.50	← = B4*1.09
6	4	$141.16	← = B5*1.09
7	5	$153.86	← = B6*1.09

The rows in the table are recursive. The balance in the second row is 1.09 times larger than the balance in the first row; the balance in the third row is 1.09 times larger than the balance in second row; more generally stated, the balance in each row (except for the first row) is larger than the balance in the previous row by a factor of 1.09. The following graph shows the growth of the account after each year for the 18 years the interest would be paid.



The balance in the bank account each year could also be expressed by the formula $y=100(1.09)^x$, where x is the number of years the account has been open and y is the balance. Check if this function has the value of $153.86 when $x=5$. (This value is indicated in the table as the balance for year 5.)



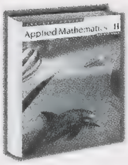
```
100(1.09)^5
            153.8623955
```

THE EXPONENTIAL FUNCTION

An **exponential function** is any function in which the independent variable occurs in the exponent. Here are a few examples:

- $y=2^x+3$
- $y=200(0.5)^{10-x}$
- $y=2(1.6)^x-9$

Such functions can describe a variety of situations, including the effects of compound interest on the balance of a bank account (as in the example given). All exponential functions have curvature to their graphs. None are linear.



Turn to page 64 of the textbook and read the opening paragraphs of Tutorial 2.1, "Exponential Graphs."

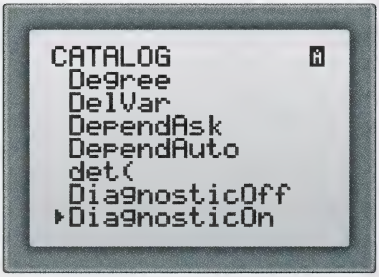
In "Investigation 1: Insect Colonies" you will need an understanding of the laws of exponents. You will also need to select appropriate window settings when you use your graphing calculator. Knowing the domain and range helps when entering the proper window settings.

1. Turn to page 64 of the textbook and answer exercises 1, 2.b., and 2.c. of "Practise Your Prior Skills."

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 63–64.

Also, in "Investigation 1: Insect Colonies" you will need to determine the exponential function that best fits a set of data. The correlation coefficient, *r*, is a measure of how close the data is represented by a function. The calculator displays the value of the square of the correlation coefficient if "DiagnosticOn" is activated from the CATALOG menu. This squared value is represented by r^2 on the screen. The closer r^2 is to 1, the better the function fits the data.

To activate DiagnosticOn so r^2 is displayed whenever you are finding the best-fit function, first press 2nd [CATALOG]. Then scroll down to "DiagnosticOn," and press ENTER twice.



```
CATALOG           A
 Degree
 DelVar
 DependAsk
 DependAuto
 det(
 DiagnosticOff
▸DiagnosticOn
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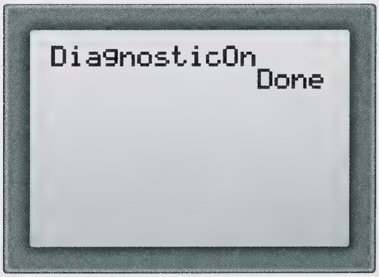
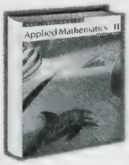


```
DiagnosticOn
            Done
```

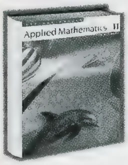


2. Turn to pages 64 and 65 of the textbook and answer exercises 1 to 4 of "Investigation 1: Insect Colonies." **Remember:** Turn on Plot 1 in the STAT PLOTS window.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 64–65.

In previous mathematics courses, you determined and drew a line of best fit for a set of data. But what if the data is not linear? In this case, you will need to determine and draw an equation of a curve that best fits the data.

Example



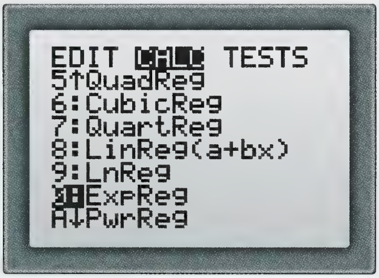
Turn to page 65 of the textbook and complete exercise 5 of "Investigation 1: Insect Colonies."

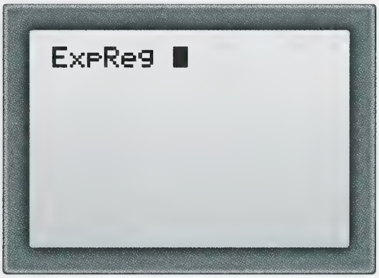
Solution

Step 1: Select the Exponential Regression feature.

Select the CALC menu.

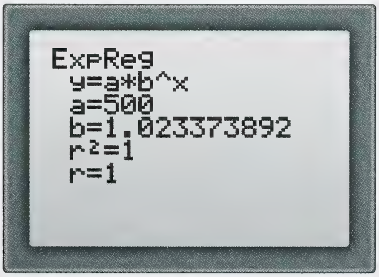
STAT ▶ 0 (0:ExpReg)





```
ExpReg ▮
```

Step 2: Press ENTER to determine the equation of best fit.



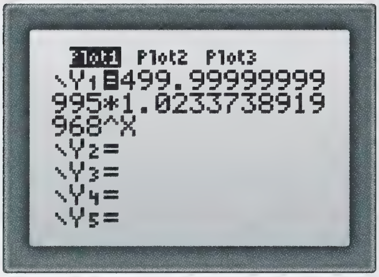
```
ExpReg
 y=a*b^x
 a=500
 b=1.023373892
 r²=1
 r=1
```

The equation of best fit for the data is $y = 500 \times 1.023\,373\,892^{x}$.

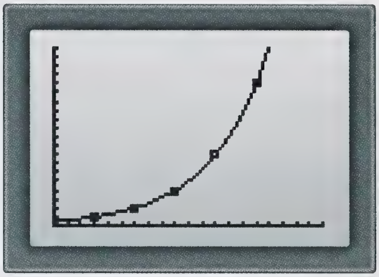
To graph the regression equation on the scatterplot, simply press the following.

Select the EQ menu.

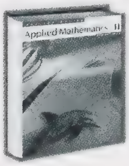
Y= VARS 5 (5:Statistics…) ▶ ▶ 1 (1:RegEQ) GRAPH



```
Plot1 Plot2 Plot3
\Y1=499.99999999
995*1.0233738919
968^X
\Y2=
\Y3=
\Y4=
\Y5=
```



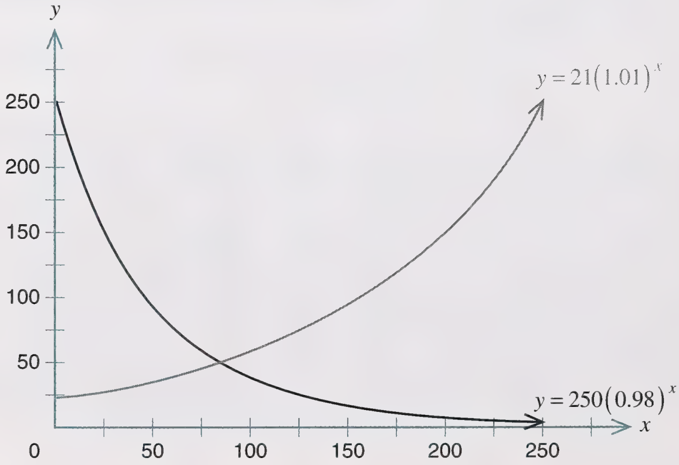
Note: The equations look slightly different because 499.999… was rounded to 500 when the exponential regression equation was determined.



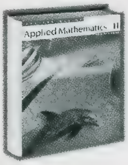
3. Turn to page 65 of the textbook and answer exercise 6 of “Investigation 1: Insect Colonies.”

Compare your response with the suggested answer in the Appendix, Activity 1, pages 65–66.

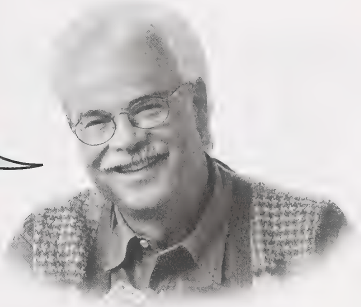
You found that a curve of the form $y = ab^x$, which is exponential, can fit data closely with suitable values for a and b. With different values for a and b, this type of curve can show a trend quite different from that in Investigation 1. Compare the following graphs of $y = ab^x$ to see the effect of changing a and b.



The value of a affects where the graph intersects the vertical axis and how much it stretches away from the horizontal axis. You can see that when b is greater than 1, the curve shows what is commonly considered to be exponential growth. When b is less than 1, the curve shows a "decay"—a decline at an ever decreasing rate towards 0.

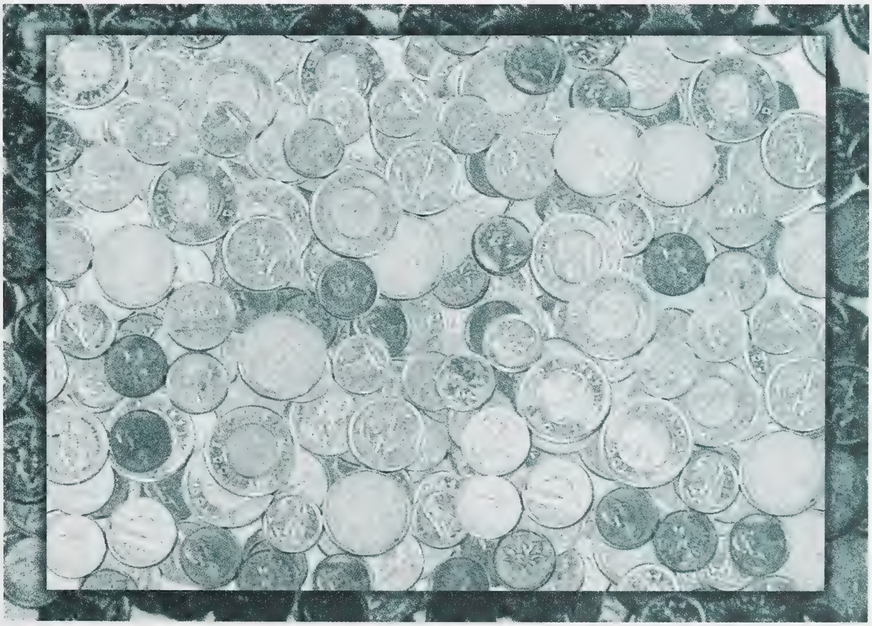


To find out why the term *decay* is used to define a decreasing exponential curve, turn to page 69 of the textbook and read the paragraph preceding "Example 2: Radioactive Decay."

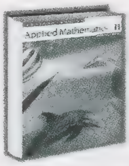


If you are having trouble picturing the decay of a radioactive isotope, imagine having a jar full of coins. Think of the coins as the atoms of an isotope. Suppose, after emptying the jar onto the floor, you removed half the coins and returned the remaining coins to the jar. Emptying the jar and refilling it with half the number of coins is like the isotope going through a half-life. Suppose you repeated the procedure over and over, each time returning only half the coins to the jar.

Graphing the number of coins in the jar with respect to the number of times you emptied the jar onto the floor is like graphing the mass of an isotope with respect to the number of half-lives it has gone through. Just as the data points of the remaining coins follow a downward sloping curve, so does the curve for a decaying radioactive isotope.



The temperature change of a cooling object also shows a trend similar to a decay curve. When something hot is left to cool, you expect it to cool quickly at first and continue cooling at an ever-slowing rate as the object approaches room temperature. Knowing this, you can use an exponential curve as the curve of best fit for the scatterplot of the temperature of an object cooling.



Turn to pages 66 and 67 of the textbook and preview "Investigation 2: Cooling Temperatures." This investigation involves recording the drop in temperature of a hot thermometer after it is plunged into crushed ice. If you are using a CBL unit with a temperature probe, follow the instructions in exercises 1 and 4 only. If you are using a thermometer, follow the instructions in exercises 1 and 2 only. **Note:** For exercise 2, take temperatures every 5 s rather than every 10 s for better results.

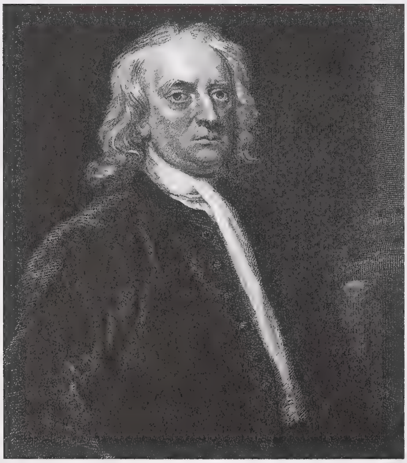
If you do not have a CBL with a temperature probe or a thermometer, use the following data and enter it into lists L1 and L2 of your graphing calculator.

Time (s)	0	5	10	15	20	25	30	35	40	45	50	55	60
Temperature (°C)	94	52	26	16	11	8	6	5	4	4	3	3	3

4. Answer exercise 5 of "Investigation 2: Cooling Temperatures" on page 67 of the textbook.

5. Answer exercise 2 of "Discussing the Ideas" on page 70 of the textbook.

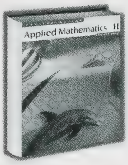
Compare your responses with the suggested answers in the Appendix, Activity 1, pages 67–68.



Sir Isaac Newton

In the preceding investigation, you used the exponential function to model the temperature trend of a cooling substance. You found that the cooling curve is very similar to a decay curve.

Approximately 300 years ago, Sir Isaac Newton developed the Law of Cooling. Newton stated that the rate at which a hot body cools is proportional to the difference between the temperature of the body at that moment and the temperature of its surroundings. Based on this proportionality, Newton used mathematics to restate the Law of Cooling as a function involving an exponential expression.



6. Turn to page 74 of the textbook and answer exercise 8 of "Exercises: Extending Your Thinking." **Note:** Let x represent the time (in seconds) on the calculator. Set $\text{Xmin} = 0$, $\text{Xmax} = 500$, and $\text{Xscl} = 50$. This allows for about 5 minutes of cooling time.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 68–69.

EXPONENTIAL GROWTH

Population growth can be modelled by an exponential function.

Example

Since 1990, the population of a town has been growing at an annual rate of 6%. The table on the right shows the population for the years 1990 to 1995.

Year	Population
1990	100 000
1991	106 000
1992	112 360
1993	119 102
1994	126 248
1995	133 823

a. Use the Exponential Regression feature on your calculator to determine the exponential function that best fits the data in the table.

b. Use algebraic reasoning based on the pattern in the data to determine the exponential function that represents the trend of the population growth.

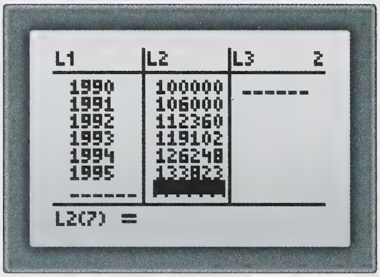
c. Graph the exponential function that best fits the data. Do not plot the data itself. Press $\boxed{\text{2nd}}$ [STAT PLOT] and turn off Plot 1.

d. According to the graph, what will be the population in 2010?

e. According to the equation in part b., what will be the population in 2010?

Solution

a. **Step 1:** Enter the data into L1 and L2.

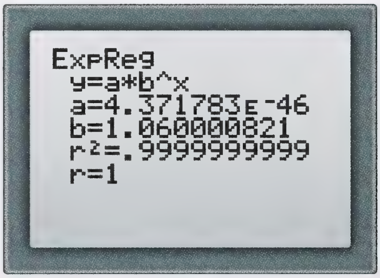


```
L1      |L2      |L3      2
 1990   | 100000 | ------
 1991   | 106000 |
 1992   | 112360 |
 1993   | 119102 |
 1994   | 126248 |
 1995   | 133823 |
 ------ |        |
L2(7) =
```

Step 2: Determine the exponential function that best fits the data.

Select the CALC menu.

STAT ▶ 0 ENTER



```
ExpReg
 y=a*b^x
 a=4.371783E-46
 b=1.060000821
 r²=.9999999999
 r=1
```

"E⁻46" represents "$\times 10^{-46}$."

b.

Year	Population
1990	100 000
1991	106 000
1992	112 360
1993	119 102

Pattern

×1.06

×1.06

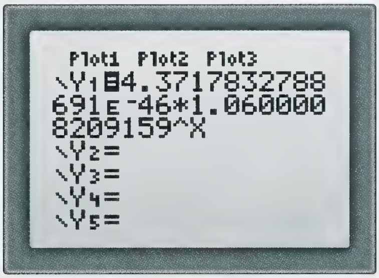
×1.06

The pattern for the population, *y*, is $y = 100\,000 \times 1.06^{t}$, where *t* is the number of years after 1990. If you replace *t* with $x - 1990$, the equation becomes $y = 100\,000 \times 1.06^{(x-1990)}$, where *x* is the calendar year.

c. **Step 1:** Insert the best-fit exponential function (from part a.) as Y_1 in the equation editor.

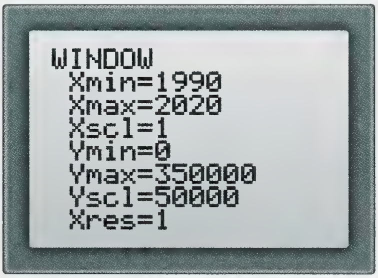
Select the EQ menu.

Y= VARS 5 ▶ ▶ 1

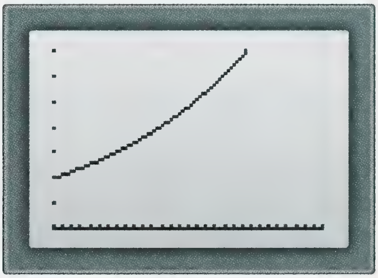


```
 Plot1  Plot2  Plot3
\Y1■4.3717832788
691E-46*1.060000
8209159^X
\Y2=
\Y3=
\Y4=
\Y5=
```

Step 2: Using the following window settings, press GRAPH.

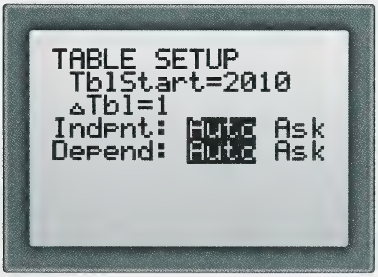


```
WINDOW
 Xmin=1990
 Xmax=2020
 Xscl=1
 Ymin=0
 Ymax=350000
 Yscl=50000
 Xres=1
```



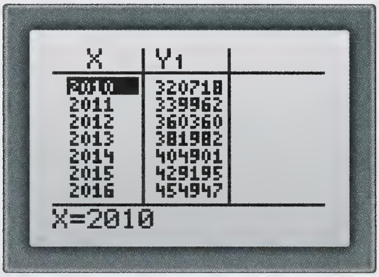
d. **Method 1: Using the Table of Values**

Step 1: Set up the table of values so it starts at year 2010. Press 2nd [TBLSET], and set up the table of values as follows.



```
TABLE SETUP
 TblStart=2010
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

Step 2: Press 2nd [TABLE], and read the y-value for $x = 2010$.



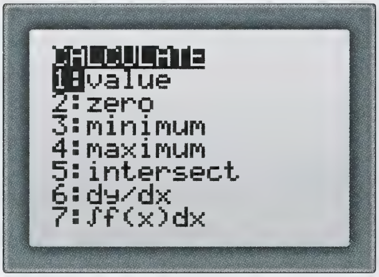
X	Y₁
2010	320718
2011	339962
2012	360360
2013	381982
2014	404901
2015	429195
2016	454947

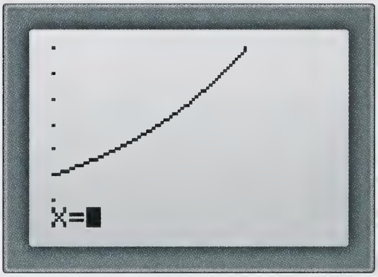
X=2010

According to the table of values, the population in 2010 will be approximately 320 718.

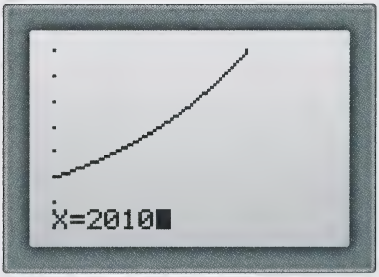
Method 2: Using the Value Feature

Step 1: Press 2nd [CALC] to access the CALCULATE menu. Then press 1 (1:value).

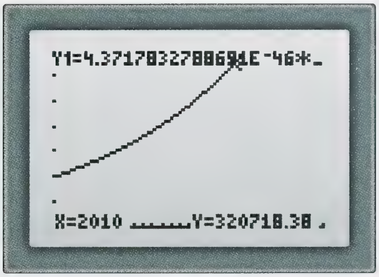




Step 2: Enter the year (2010).



Step 3: Press ENTER.



```
Y1=4.37178327886914E-46*_
X=2010        Y=320718.38
```

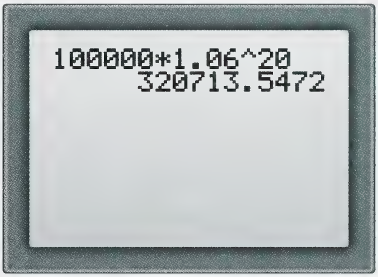
According to the graph, the population in 2010 will be approximately 320 718.

e. $y = 100\ 000 \times 1.06^{(x-1990)}$

$= 100\ 000 \times 1.06^{(2010-1990)}$

$= 100\ 000 \times 1.06^{20}$

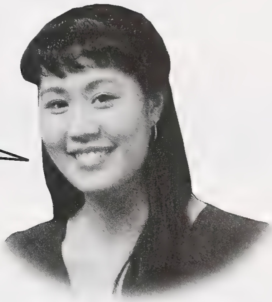
$\doteq 320\ 714$

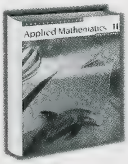


```
100000*1.06^20
       320713.5472
```

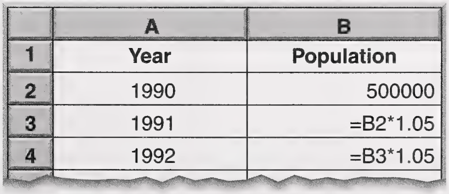
According to the equation, the population in 2010 will be approximately 320 714.

Note that the population determined by the equation is very close to the projection based on the exponential regression curve.



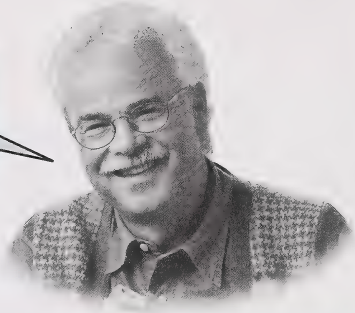


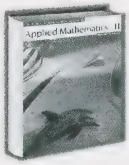
Turn to pages 67 to 69 of the textbook and work through "Example 1: Population Growth." For part a., if you are using a spreadsheet, you may want to set up the formulas as follows.



	A	B
1	Year	Population
2	1990	500000
3	1991	=B2*1.05
4	1992	=B3*1.05

Using the Regression feature and algebraic reasoning gives you equivalent functions.





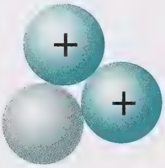
7. Turn to page 72 of the textbook and answer exercises 2.a. to 2.c. of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 69–71.

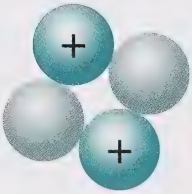
EXPONENTIAL DECAY

The radioactive decay of isotopes led to the use of the term *decay* to refer to any kind of decrease that can be modelled by an exponential function.

What are isotopes? Isotopes of an element have the same number of protons but a different number of neutrons. Helium-3 and helium-4 are examples of isotopes. Helium-3 has one neutron in its nuclei, and helium-4 has two neutrons.



Helium-3



Helium-4

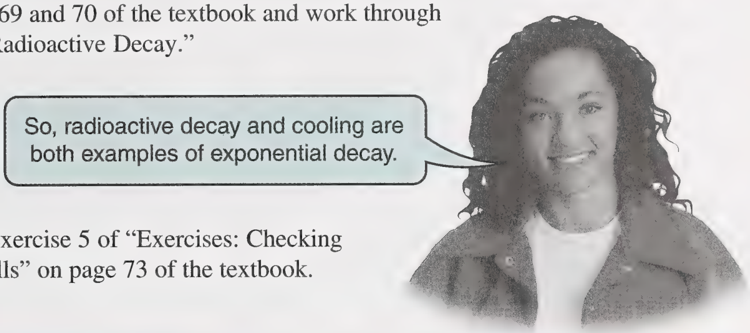
Although helium-3 and helium-4 are both stable, some isotopes of other elements are unstable.

Unstable isotopes of an element "break down" or decay spontaneously into isotopes of other elements. Because the decay is accompanied by radiation, the change is referred to as radioactive decay and the unstable isotope is described as radioactive. Which atoms break down is not predictable, but the time required for a given **fraction** of a sample to decay is predictable.

The time for one-half the mass of an unstable isotope to decay is called the **half-life** of the isotope. Half-lives can vary from fractions of a second for one isotope to billions of years for another. Each unstable isotope has a characteristic half-life, which is independent of the sample size.

The change in mass of an unstable isotope over time can be modelled by an exponential function.

Turn to pages 69 and 70 of the textbook and work through "Example 2: Radioactive Decay."


So, radioactive decay and cooling are both examples of exponential decay.

8. Answer exercise 5 of "Exercises: Checking Your Skills" on page 73 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 71–72.

Applications

A single general equation can be written to apply to both exponential growth and decay.

Turn to page 70 of the textbook and read the paragraph preceding "Discussing the Ideas."

9. Turn to page 71 of the textbook and answer exercises 4 and 5 of "Discussing the Ideas."

10. Turn to pages 71 and 72 of the textbook and answer exercises 1, 3, and 4 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 72–79.

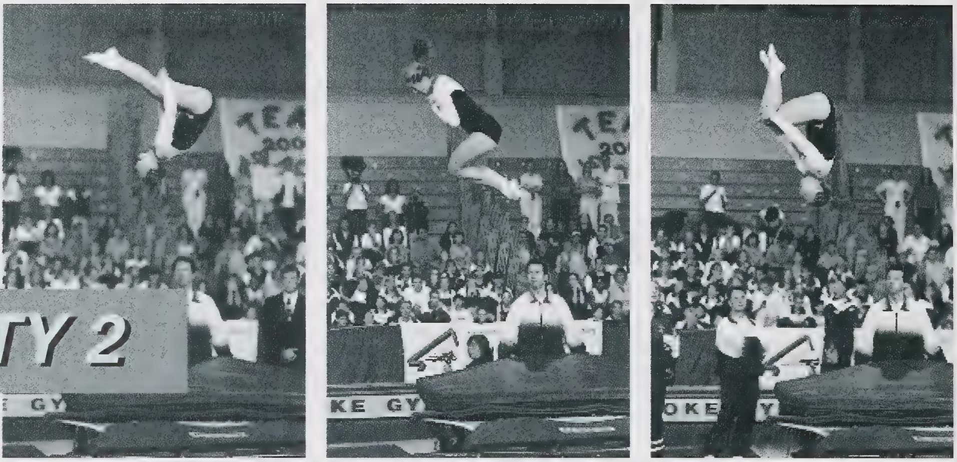
LOOKING BACK

In this activity you plotted and described exponential data, used exponential regression to determine the equation that best fits a set of data, and applied exponential functions to growth and decay problems.

11. Turn to page 74 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 1, page 80

ACTIVITY 2

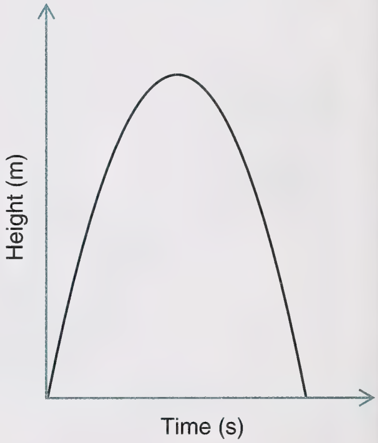


GRACE CHIU

Quadratic Graphs

When jumping on a trampoline, you go up for some time, stop momentarily, and come down to the surface again. If someone made a graph to show your height during a jump with respect to time, the graph would be a quadratic graph (like the one on the right).

Quadratic graphs can be used to represent data relating to any body or object rising and falling under the influence of gravity. Quadratic graphs may also apply to data related to maximizing revenue.

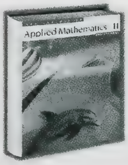


The Height of One Jump on a Trampoline

Turn to page 75 of the textbook and read the first paragraph of Tutorial 2.2, "Quadratic Graphs" and the first paragraph of "Investigation 1: What Goes Up."

1. Answer exercises 1 to 5 of Part B of "Investigation 1: What Goes Up" on pages 76 and 77 of the textbook. **Note:** If you have access to a CBR or CBL unit, you may wish to do Part A as well.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 80–82.



Turn to page 77 of the textbook and read the text preceding "Investigation 2: The Characteristics of Quadratic Graphs."

2. Based on the general form, $y = ax^2 + bx + c$, give the values of *a*, *b*, and *c* in each of the following quadratic functions.

a. $y = 5x^2 + 2x + 3$

b. $y = 3x^2 - 2$

c. $y = x^2 - x + 1$

d. $y = x^2$

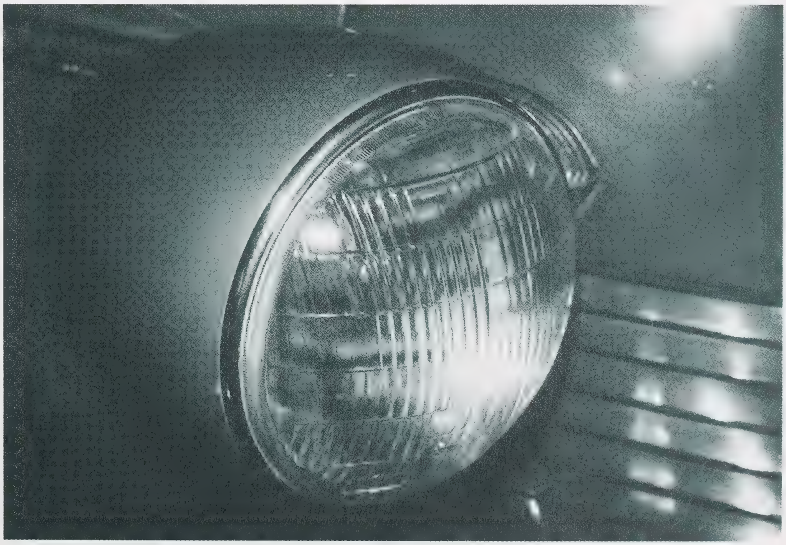
Compare your responses with the suggested answers in the Appendix, Activity 2, page 82.

The graph of a quadratic function is a parabola. The parabolic shape is used in various devices that use a reflecting surface. It's the cross section through the centre of the surface that is designed to be parabolic.

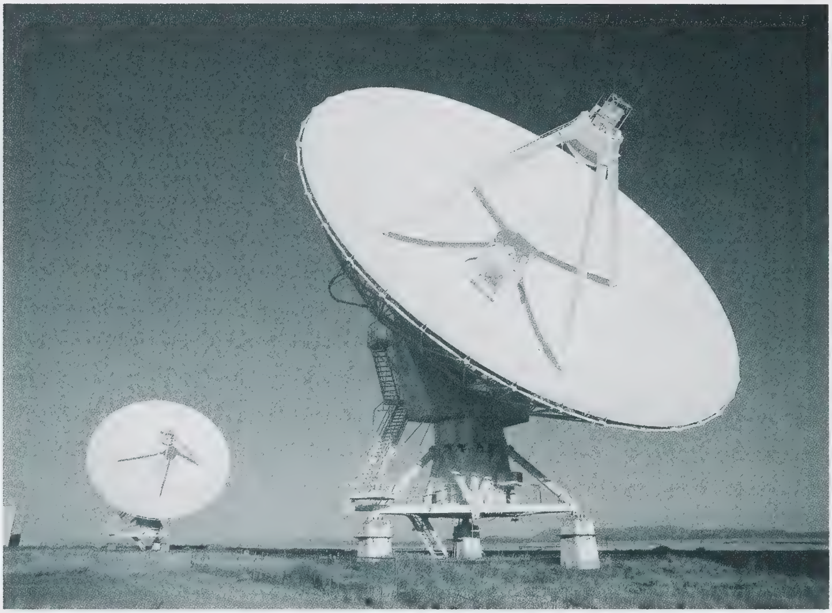
A parabolic microphone consists of a parabola-shaped reflecting surface and a microphone placed ahead of its centre. The parabolic surface concentrates sound by reflecting sound waves to the position of the microphone. Because the sound is concentrated, the microphone can catch voices ahead of the microphone more than a kilometre away. Such microphones are used in surveillance.

EFSTONSCIENCE

The reflector of a car's headlight assembly is also shaped like a parabola. The shape of the reflector is easily identified in older cars. The parabolic shape is responsible for projecting of a beam of light from the lamp filament.

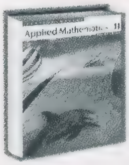


Another example of a parabolic shape is the reflecting surface of a radio telescope. Like the parabolic microphone, the surface reflects faint radio signals to a central focus where the detector is located.



The parabola you obtain when graphing a quadratic equation either has a maximum value or a minimum value. Either value can be found using your graphing calculator.

Example



Find the coordinates of the minimum value for the function $y = x^2 + x - 2$.

Solution

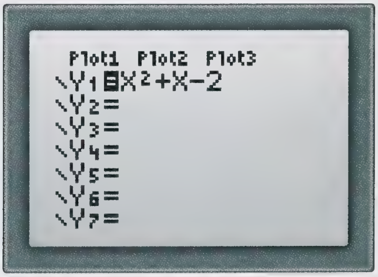
Before starting, turn off all plots in the STAT PLOTS menu.

First, graph the equation.

Step 1: Enter the equation to be graphed.

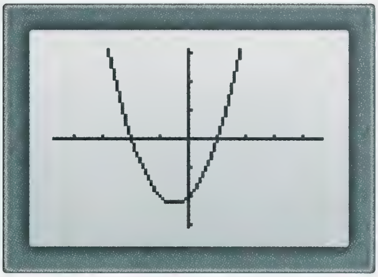
Press Y= , and clear any entries if necessary; then enter the equation $y = x^2 + x - 2$.

X,T,Θ,*n* x^2 + X,T,Θ,*n* − 2



```
Plot1 Plot2 Plot3
\Y1■X²+X-2
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Step 2: Graph the equation using preset windows settings of Zdecimal by pressing ZOOM 4 (4:Zdecimal). This sets the height and width of each pixel of the screen to a value of 0.1.

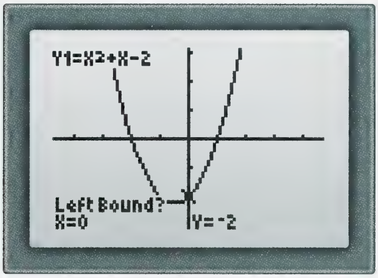


Once the graph is displayed, press WINDOW to see the value of the window variables. Then press GRAPH to return to the graph.

Now, find the coordinates of the minimum point.

Step 1: Select the minimum feature from the CALCULATE menu.

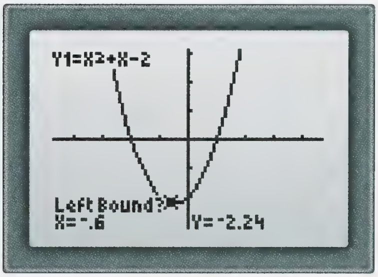
2nd [CALC] 3
(3:minimum)



The cursor appears at $(0, 2)$, and the calculator prompts you to choose the left bound. The left bound indicates the furthest point to the left that the calculator must check as a possible minimum.

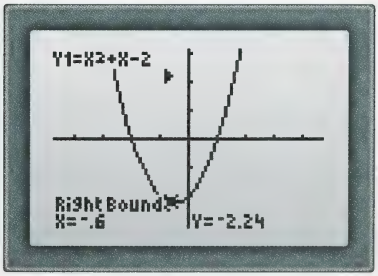
Step 2: Choose the left bound.

Using the arrow keys, move the cursor just to the left of what appears to be the lowest point of the graph.



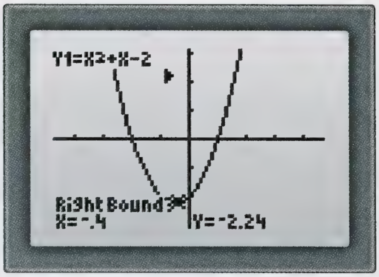
Step 3: Press ENTER to set the value for the left bound.

Now, the calculator prompts you to choose the right bound. The right bound indicates the furthest point to the right that the calculator must check as a possible minimum.



Step 4: Choose the right bound.

Using the arrow keys, move the cursor just to the right of what appears to be the lowest point of the graph.



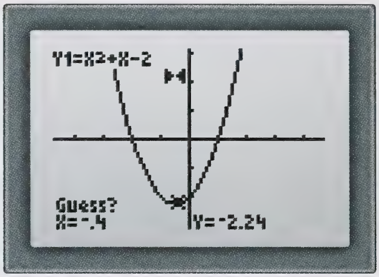
Y1=X²+X-2

Right Bound?

X=-.4 Y=-2.24

Step 5: Press ENTER to set the right bound.

Now, the calculator prompts you to enter your guess.

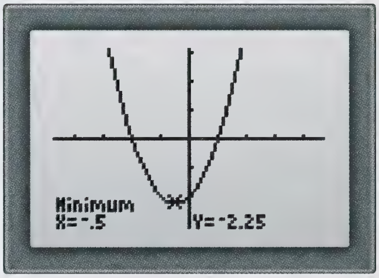


Y1=X²+X-2

Guess?

X=-.4 Y=-2.24

Step 6: Tap the left arrow key once, and press ENTER to obtain the coordinates of the minimum point.



Minimum

X=-.5 Y=-2.25

You can improve the accuracy of the calculated minimum value by using the Zoom feature and setting the left and right bounds as close to the actual minimum point as possible.

The minimum value of the function $y = x^2 + x - 2$ is $(-0.5, -2.25)$.

You will examine more about the parabola and how it relates to the quadratic equation in the next investigation.

The graph of the function in the preceding example crossed the x-axis. The x-value of any point where the graph crosses the x-axis is referred to as a **zero of the function**. Bear in mind that not all graphs of functions cross the x-axis; so, not all functions have zeros.

In the next investigation, you will use the Zero feature to find the zeros of a function. Using this feature is similar to using the Minimum feature, illustrated in the preceding example, except you are choosing a left and a right bound to find a zero, not a minimum.

Refer to "Utility 19: The CALC Feature of the TI-83 Calculator" on pages 413 and 414 of the textbook for additional help.

3. Turn to pages 77 and 78 of the textbook and complete exercise 1 of "Investigation 2: The Characteristics of Quadratic Graphs." **Note:** In exercise 1.d., use the features from the CALCULATE menu, 2nd [CALC]. Also, before using the Draw feature, press CLEAR to clear the screen.

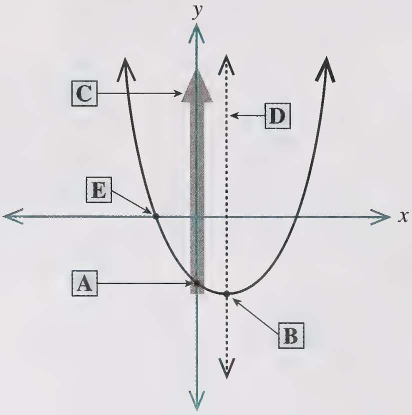
4. Answer exercises 2.i. to 2.iv. of "Investigation 2: The Characteristics of Quadratic Graphs" on page 78 of the textbook.

5. Answer exercises 3 to 8 of "Discussing the Ideas" on page 79 of the textbook.

Read the paragraph at the bottom of page 78 of the textbook; then complete the exercises that follow.

6. Match the letters in the diagram with the following terms:

- axis of symmetry
- domain
- range
- vertex
- *x*-intercept
- *y*-intercept



A. ____________________

B. ____________________

C. ____________________

D. ____________________

E. ____________________

7. Turn to pages 79 to 81 of the textbook and answer exercises 1.a., 2.a., 2.b., 3.a., 3.e., 6.a., 6.b., and 6.d. of “Exercises: Checking Your Skills.”

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 82–94.

LOOKING BACK

In this activity, you discovered that the scatterplot of some data fit on a smooth curve shaped like the graph of a quadratic function. You then graphed quadratic equations and determined the axis of symmetry, the coordinates of the vertex, the domain and range, and the intercepts.

8. Turn to page 81 of the textbook and answer “Communicating the Ideas.” You may wish to write a paragraph defining a quadratic function and describing the graph of a quadratic function using the terms given at the bottom of page 81 of the textbook.

Compare your response with the suggested answer in the Appendix, Activity 2, page 94.

ACTIVITY 3

Problem Solving Using Quadratic Functions

Promoters of events for large audiences know that the number of tickets sold is price-sensitive—raise the ticket price and fewer tickets will be sold. On the other hand, if ticket prices are set too low, the resulting revenue will not be enough to meet the expense of hosting the event, let alone to make a profit.

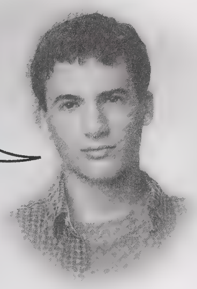
Quadratic functions can help a promoter determine the optimum ticket price for an event to maximize revenue. They can also be used to determine the position of a falling object after it is released or the height of a projectile after it is propelled upwards.

MODELLING FROM A DATA SAMPLE

In the next example, the quadratic function is not given to you directly. You must obtain it from a small data sample. There you will use the Quadratic Regression feature on your graphing calculator to get the best-fit quadratic function.

Turn to page 86 of the textbook and work through “Example 1: Ticket prices.” Remember to clear functions from the equation editor and turn on Plot 1 in the STAT PLOTS menu before you start working through the example.

The preferred method of determining the maximum is to use the Maximum feature from the CALCULATE menu, not the Trace feature.



1. Answer exercises 2 and 3 of “Exercises: Checking Your Skills” on pages 90 and 91 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 95–96.

When an outfielder throws a ball in a game of baseball, the ball itself follows a parabolic path towards the infield. Earlier, when you graphed the height of a ball thrown straight up, you also ended up with a parabola—even though the ball’s trajectory was not a parabola. Your textbook cautions you against thinking of the height-versus-time graph as a picture of the trajectory.

Turn to page 87 of the textbook and read the paragraph following “Example 1: Ticket prices”; then study the graphs at the top of page 88.

USING A FORMULA

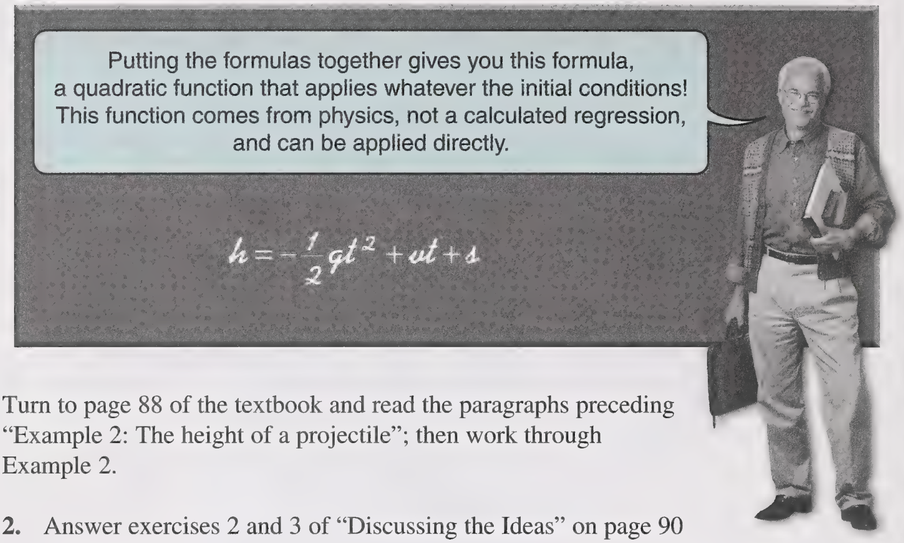
You may remember from an earlier science course that there are formulas that predict how far objects move based on the time for which they undergo acceleration. Acceleration is the rate at which speed changes. When a "heavy" object, like a rock, falls after being let go from the edge of a cliff, its height changes under the influence of the acceleration due to gravity (as shown in the following table).

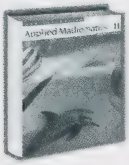
Time (s)	Height (m)
0	0
1	−4.9
2	−19.6
3	−44.1
4	−78.4
5	−122.5
6	−176.4
7	−240.1
8	−313.6

Note: Height is given in negative values because these values represent distance in a negative direction from the initial starting point.

The position of the rock, *h*, at any time, *t*, can be represented by the fomula $h = -\frac{1}{2}gt^2$, where *g* is the acceleration due to gravity $\left(9.8\ \mathrm{m/s^2}\right)$ on Earth.

You may also remember that if there is no acceleration on an object—say, a puck gliding on an air hockey table—the distance, *d*, of an object from a fixed reference point is predicted by the formula $d = vt + s$. In this formula, *v* is the initial (and constant) velocity of the object and *s* is the initial distance of the object from the reference point.


Putting the formulas together gives you this formula, a quadratic function that applies whatever the initial conditions! This function comes from physics, not a calculated regression, and can be applied directly.
$h=-\frac{1}{2}gt^2+vt+s$



Turn to page 88 of the textbook and read the paragraphs preceding "Example 2: The height of a projectile"; then work through Example 2.

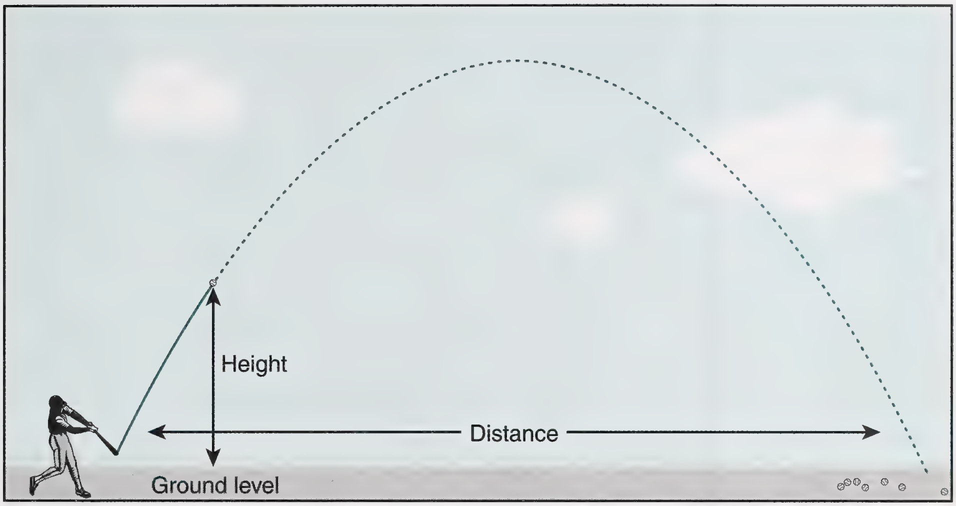
2. Answer exercises 2 and 3 of "Discussing the Ideas" on page 90 of the textbook.

3. Answer exercises 1 and 7 of "Exercises: Checking Your Skills" on pages 90 and 93 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 97–99.

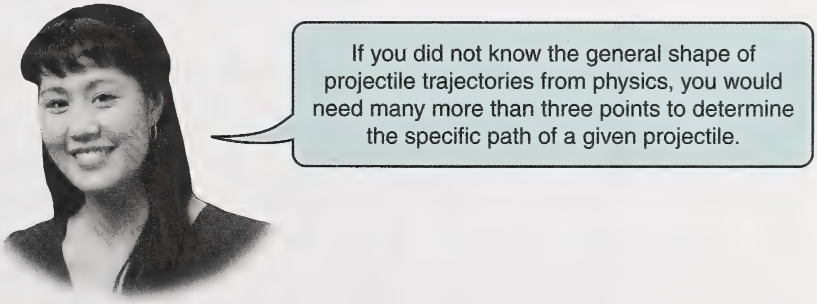
TRAJECTORY

Remember that a height-versus-time graph of an object is usually not a scale diagram of the path of the object in the air. However, you can make a graph that is a scale version of an object's path. To make such a graph, you have to graph the height of the object to its horizontal distance for its flight through the air.



There is no simple, general quadratic equation from physics that describes height as a function of distance. But physics does indicate that the path of an object thrown through the air (or the trajectory), ignoring air resistance, is a parabola.

To determine the function for the parabolic trajectory, simply fit a quadratic function to at least three known points of the trajectory.


If you did not know the general shape of projectile trajectories from physics, you would need many more than three points to determine the specific path of a given projectile.

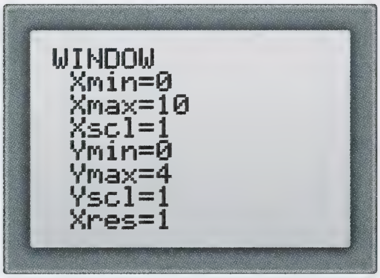
Example

A shot-put is released from a height of 1.4 m. The shot-put reaches a maximum height of 3.7 m at a horizontal distance of 4.2 m from the point of release; then it hits the ground 8.8 m from the point of release.

DAVID GREY/REUTERS

a. List three points on the trajectory.

b. Using the following window settings, make a scatterplot using the three data points from part a.



```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=4
 Yscl=1
 Xres=1
```

c. Determine the height as a function of the horizontal distance travelled. Base your function on the quadratic equation that best fits the data points.

d. Graph the equation in part c. over the data points.

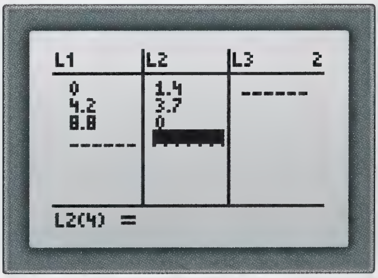
Solution

a. The first known point is $(0, 1.4)$, which is the point where the shot-put is released. The second known point is $(4.2, 3.7)$, which is the point where the shot-put reaches a maximum height. The third known point is $(8.8, 0)$, which is the point where the shot-put hits the ground.

b. **Step 1:** Clear any equation from the equation editor, and clear all lists.

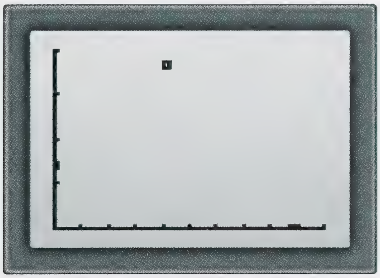
Step 2: Enter the data in L1 and L2.

Enter the horizontal distances in L1 and the heights in L2.



```
L1     |L2     |L3     2
0      |1.4    |------
4.2    |3.7    |
8.8    |0      |
------ |       |
L2(4) =
```

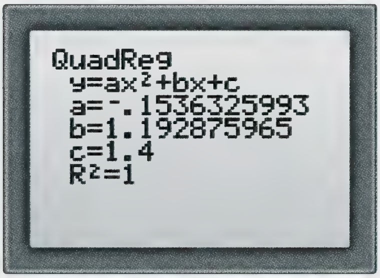
Step 3: Press GRAPH to show the scatterplot. **Note:** Use the square mark. The other markers are not visible when plotted on the axes.



c. Determine the quadratic equation that best fits the data.

Select the CALC menu.

STAT ▶ 5 (5:QuadReg)

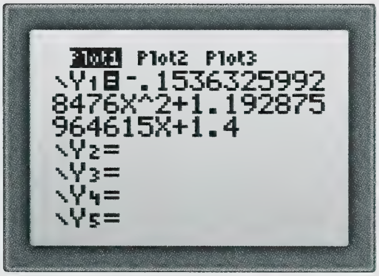


```
QuadReg
 y=ax²+bx+c
 a=-.1536325993
 b=1.192875965
 c=1.4
 R²=1
```

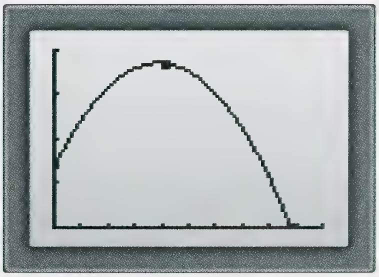
d. Place the function into the equation editor, and graph the function.

Select the EQ menu.

Y= VARS 5 (5:Statistics…) ▶ ▶ 1 (1:RegEQ) GRAPH



```
Plot1 Plot2 Plot3
\Y1■-.1536325992
8476X^2+1.192875
964615X+1.4
\Y2=
\Y3=
\Y4=
\Y5=
```



The quadratic equation that best fits the data is $y = -0.15x^2 + 1.19x + 1.4$.

A trajectory is the actual path followed by an object. To graph the path of the shot-put in the previous example as a trajectory, you need to set the window settings so that height to distance is proportional.

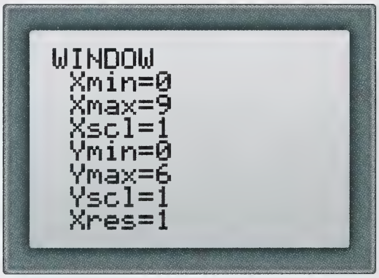
Notice that Xscl and Yscl are both equal to 1, but the tick marks are father apart on the vertical axis. This indicates that the graph, as a picture of the trajectory, is stretched too much in the vertical direction. The shot-put really does not go so high in comparison to the distance of the throw.

The screen of the TI-83 graphing calculator displays its image by turning on and off small square dots called pixels. The screen has 62 rows of 94 pixels. When you choose window settings for your graphs, you are telling the calculator how to spread the image amongst these dots. To get an image that corresponds to most people's view of reality, the horizontal and vertical number of pixels for the same length has to be the same. This relationship for making things close to reality says that you must use carefully chosen numbers for the window settings.

How do you choose these values? To show you, let $\Delta \text{X} = \text{Xmax} - \text{Xmin}$ and $\Delta \text{Y} = \text{Ymax} - \text{Ymin}$. If you make sure that ΔX is close to 1.5 times the size of ΔY or that ΔY is $\frac{2}{3}$ times the size of ΔX, your graph will not be noticeably distorted.

Step 1: Make the windows settings reflect the ratio of the screen dimensions.

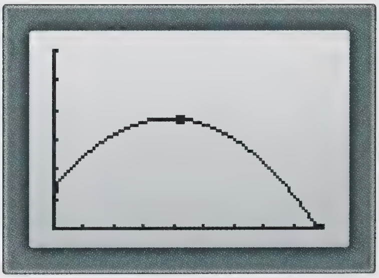
Set $\text{Xmin} = 0$ and $\text{Xmax} = 9$ so the domain of the function is included on the horizontal axis. The number 9 is also convenient for determining the length of the vertical axis.



```
WINDOW
 Xmin=0
 Xmax=9
 Xscl=1
 Ymin=0
 Ymax=6
 Yscl=1
 Xres=1
```

With the X's set, $\text{Ymax} - \text{Ymin}$ should then be $\frac{2}{3} \times 9$, which is 6. Set $\text{Ymin} = 0$ and $\text{Ymax} = 6$.

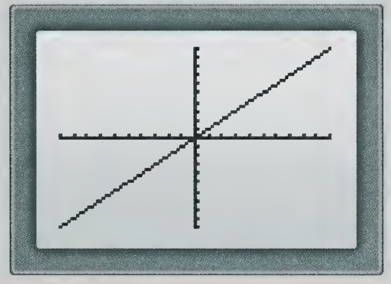
Step 2: Press GRAPH to display the trajectory of the shot-put.

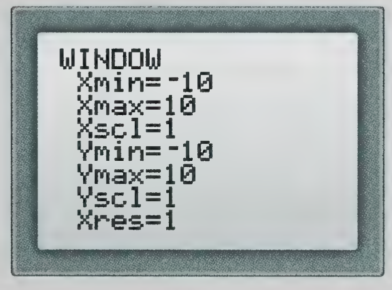


The resulting graph is the same shape as the trajectory of the shot-put. The shot-put is normally released at an angle of 40°. This is shown at the final graph, but not in the one for part d. of the preceding example.

You may want to do more investigation. Enter the equation $Y = X$. Graph it by pressing ZOOM 6. Check the window settings. These settings are for the standard window. Then press ZOOM 5. This gives the 45° line that you expect from the linear function $y = x$ when there is no distortion.

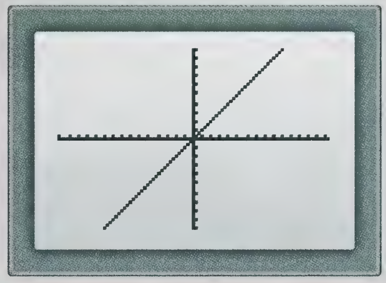
ZOOM 6

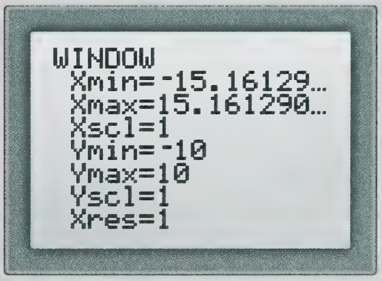




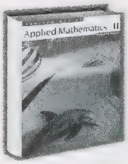
```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

ZOOM 5

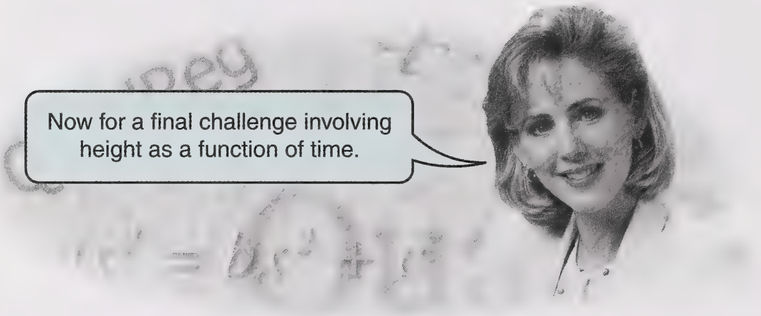




```
WINDOW
 Xmin=-15.16129…
 Xmax=15.161290…
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```



4. Turn to pages 93 and 94 of the textbook and answer exercises 9 and 10 of "Exercises: Checking Your Skills." For exercise 9, assume the maximum height is reached in the middle of the trajectory.

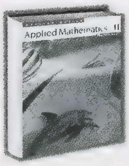

Now for a final challenge involving height as a function of time.

5. Turn to page 94 of the textbook and answer exercise 12 of "Exercises: Extending Your Thinking." **Hint:** Adapt the formula $h = -\frac{1}{2}gt^2 + vt + s$ by substituting the value for v and the value for s that apply to a stone dropping from the time it is let go at ground level.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 99–104.

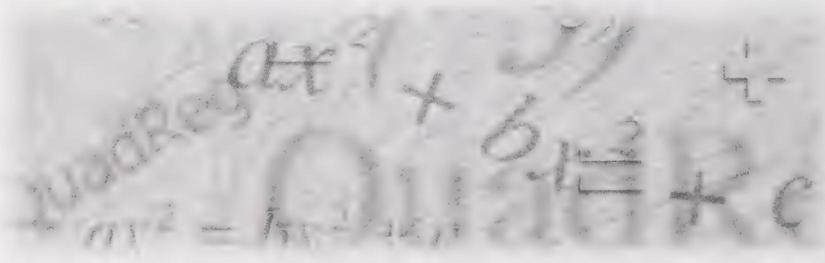
LOOKING BACK

In this activity you solved problems using quadratic functions. You focused on the use of quadratic functions to find the optimum price for an item and to describe the way an object moves under the influence of gravity. You also looked at the difference between height-time graphs and height-distance graphs.



6. Turn to page 94 of the textbook and answer "Communicating the Ideas."

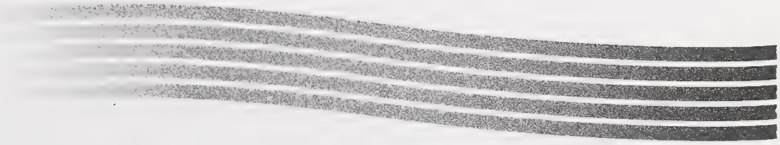
Compare your response with the suggested answer in the Appendix, Activity 3, page 104.



ACTIVITY 4

Solving Non-Linear Equations

When police investigate a traffic violation or an accident, skid marks can provide valuable information about the speed of the vehicle before the brakes were applied. Solving a non-linear equation that relates speed to the length of a skid mark can determine the original speed of the vehicle.



In this activity you will solve non-linear equations using your calculator; solve systems of equations, where one equation is linear and another is non-linear; and model real-world situations with the most suitable non-linear equation.

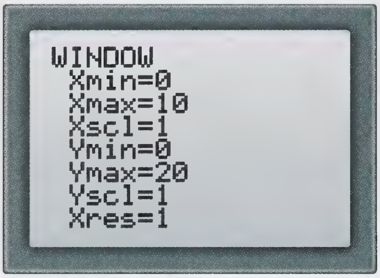
METHODS OF SOLVING EQUATIONS

In order to solve $5x + 4 = 14$, you have to find the value of x that satisfies the equation. You can probably solve this equation just by trying to fit a number or two. Otherwise, you can easily solve it algebraically. Did you know that you can solve this equation (and other equations like it) by graphing it on your graphing calculator?

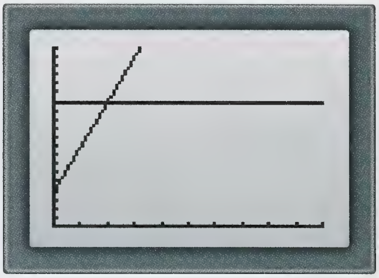
Method 1: Using Intersecting Graphs

The equation can be solved by determining where graphs of two functions intersect. In this case, the two functions are $y = 5x + 4$ and $y = 14$.

Step 1: Graph $y = 5x + 4$ as Y_1 and $y = 14$ as Y_2.



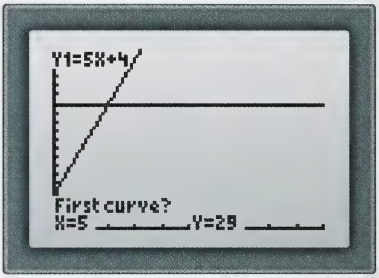
```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=20
 Yscl=1
 Xres=1
```



Step 2: Select the Intersect feature from the CALCULATE menu.

2nd [CALC] 5 (5:intersect)

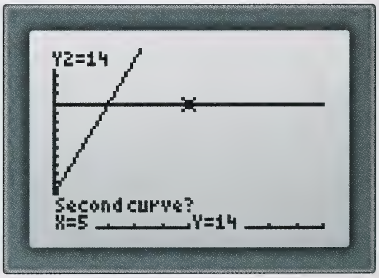
The cursor appears at $(5, 29)$, and the calculator asks you if you wish to choose the graph of $y = 5x + 4$ as the first curve.



```
Y1=5X+4
First curve?
X=5          Y=29
```

Step 3: Press ENTER to choose the first curve.

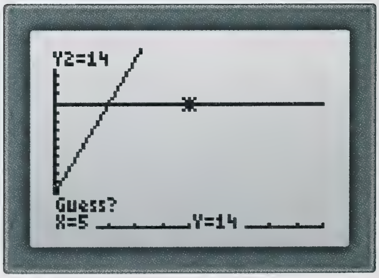
The calculator now asks if you wish to choose the graph of $y = 14$ as the second curve.



```
Y2=14
Second curve?
X=5          Y=14
```

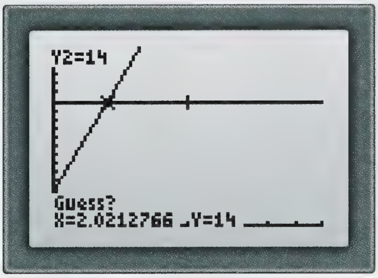
Step 4: Press ENTER to choose the second curve.

The calculator now asks you to enter a guess.

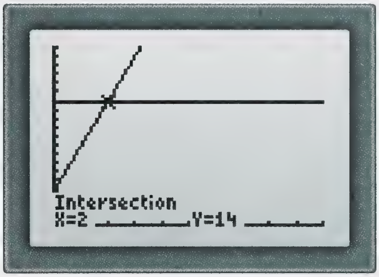


```
Y2=14
Guess?
X=5          Y=14
```

Step 5: Using the arrow keys, move the cursor so it is close to the point of intersection; then press `ENTER` to find the point of intersection.



```
Y2=14
Guess?
X=2.0212766  Y=14
```



```
Intersection
X=2          Y=14
```

The graphs intersect at $x = 2$. Therefore, the solution to the original equation is $x = 2$.

Method 2: Using *x*-Intercepts

You can also solve the equation by transforming the equation into a function and finding the zeros.

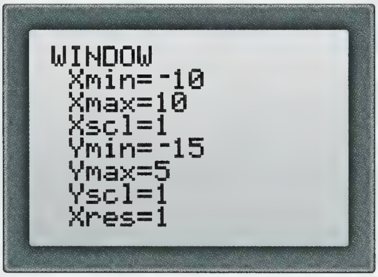
$$5x + 4 = 14$$

$$5x + 4 - 14 = 14 - 4$$

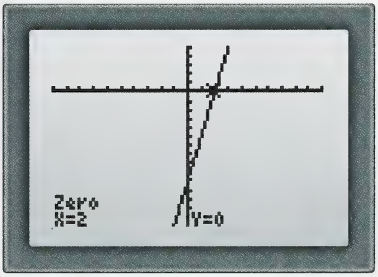
$$5x - 10 = 0$$

$\therefore\ y = 5x - 10$

Using the given window settings, graph the function and find the zero(s).



```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-15
 Ymax=5
 Yscl=1
 Xres=1
```



```
Zero
X=2          Y=0
```

The intercept $x = 2$ is the solution to the original equation.

1. Verify that $x = 2$ is a solution to $5x + 4 = 14$.

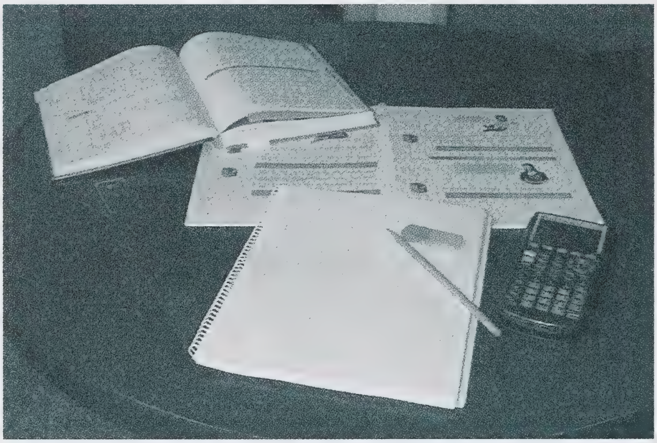
Compare your response with the suggested answer in the Appendix, Activity 4, page 104.

If these two methods work with non-linear equations, they may be handy when solving equations that are not easily solved algebraically or by inspection. Do you think these methods will work with non-linear equations? See for yourself in the next investigation.

2. Turn to page 95 of the textbook and answer exercises 1 to 3 of “Investigation 1: Intersecting Graphs.” Exercises 1 to 3 use Method 1 of solving a non-linear equation.

3. Turn to page 95 of the textbook and answer exercises 4 to 8 of “Investigation 1: Intersecting Graphs.” Here, you will use Method 2 of solving a non-linear equation.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 105–110.



Turn to pages 98 and 99 of the textbook and work through “Example: Solve a non-linear equation using a graphing calculator.” **Note:** The standard window settings will work fine if you are using the features from the CALCULATE menu.

4. Answer exercises 1, 3, and 7 of “Discussing the Ideas” on page 99 of the textbook.

5. Answer exercises 1.c., 3.a., and 3.b. of “Exercises: Checking Your Skills” on page 100 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 110–112.

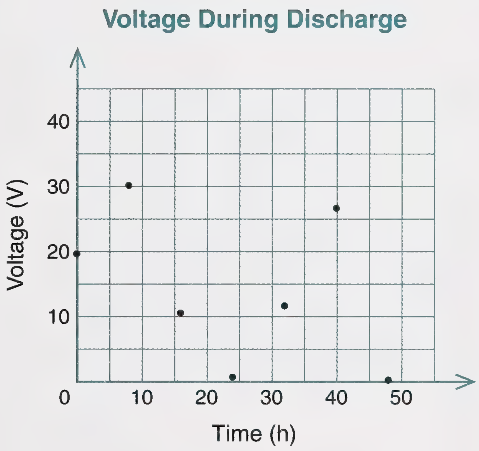
MODELLING WITH REGRESSION FUNCTIONS

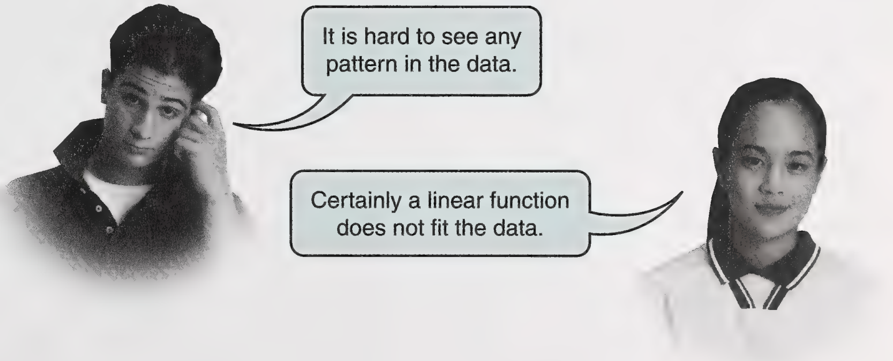
Determining a function that shows the trend of a sample of data is to **model** the data.

When you determined the line of best fit for a set of data in earlier mathematics and science courses, you were modelling the data with a function. In Activity 1 you used exponential functions to model data representing exponential growth or decay. In Activity 2 you found that quadratic functions could model data related to objects moving under the influence of gravity.

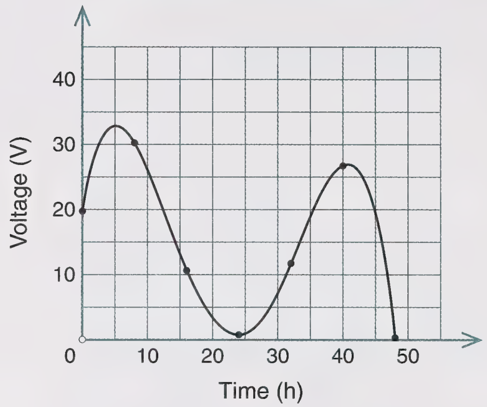
There are other functions that may apply to different situations. For example, a scientist measures the voltage from an electrochemical reaction as it discharges. Here are the results.

Time (h)	Voltage (V)
0	19.7
8	30.2
16	10.6
24	0.74
32	11.7
40	26.7
48	0.3


Voltage During Discharge


It is hard to see any pattern in the data.
Certainly a linear function does not fit the data.

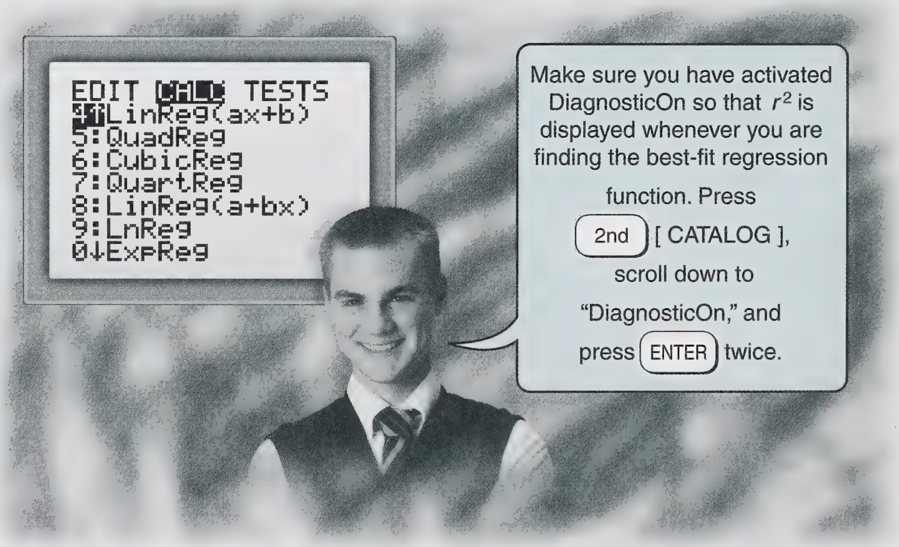
The scientist tried fitting several functions to the curve and found that a quartic function modelled the data surprisingly well (as shown in the following graph).


Voltage During Discharge

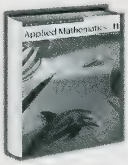
The algebraic model for this data turned out to be the following:

$$y = -0.0003x^4 + 0.0267x^3 - 0.7472x^2 + 5.7244x + 19.703$$

You, too, can model data and discover patterns using your graphing calculator. Your graphing calculator gives you several regression functions from which to choose when modelling data. Recall that you press STAT ▶ to access these functions.



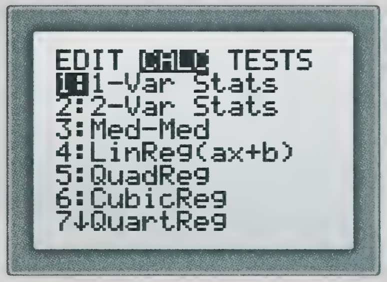
Make sure you have activated DiagnosticOn so that r^2 is displayed whenever you are finding the best-fit regression function. Press 2nd [CATALOG], scroll down to "DiagnosticOn," and press ENTER twice.



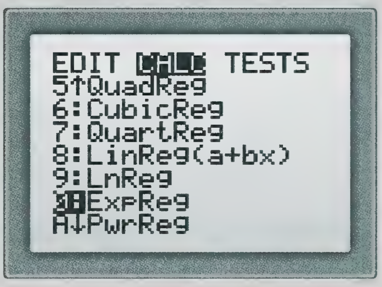
6. Turn to pages 96 and 97 of the textbook and work through at least one part (A, B, or C) of “Investigation 2: Modelling Real Situations Using Regression Techniques.” Use $\text{Xmin} = 0$ and $\text{Ymin} = 0$. Set Xmax, Xscl, Ymax, and Yscl for each as the data requires.

You may want to refer to the general steps involved in using regression functions as you do question 6.

Step 1: Go to the menu presenting the regression functions by pressing STAT and selecting the CALC menu.



Step 2: From the CALC menu, select the regression function you think is best. The various regression functions all end in “Reg.”



Step 3: Calculate the constants for the regression function by pressing ENTER twice. Then wait for the general equation and its constants to appear.

Step 4: Place the best-fit exponential function into the equation editor.

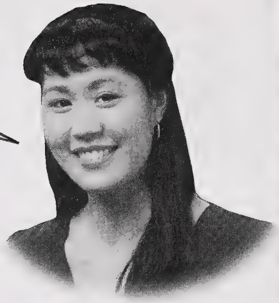
Clear any equations in the equation editor. Then press Y= VARS 5, select the EQ menu, and press 1 (1:RegEQ).

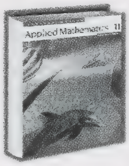
You should see the function in the equation editor.

Step 5: Press GRAPH to graph the regression equation.

If you need more detailed information about using regression equations, take time to work through “Utility 18: Determine a Regression Equation for Data Using the TI-83” on pages 411 and 412 of the textbook.

If possible, share your results in exercise 6 with another student taking Applied Mathematics 20.



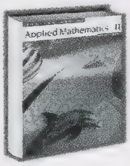


7. Turn to pages 97 and 98 of the textbook and answer exercise 2, 3, or 4 of "Investigation 2: Modelling Real Situations Using Regression Techniques," depending on which part of the investigation you completed.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 112–121.

MORE APPLICATIONS

In applying non-linear functions, you may have to model data or solve functions. Sometimes you have to do both. Applying the equations in solving problems involves choosing what to do.



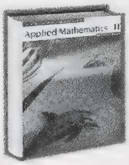
8. Turn to pages 100 to 102 of the textbook and answer exercise 4, 5, and 10 of "Exercises: Checking Your Skills."

9. Turn to page 108 of the textbook and answer exercise 6 of Part B of "What Should I Be Able To Do?"

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 122–126.

LOOKING BACK

In this activity, you modelled data with non-linear functions and solved non-linear equations using your graphing calculator.



10. Turn to page 103 of the textbook and answer "Communicating the Ideas."

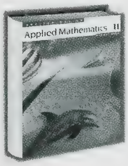
Compare your response with the suggested answer in the Appendix, Activity 4, page 127.

Follow-up Activities

This module dealt with Chapter 2: Non-Linear Functions in the *Addison-Wesley Applied Mathematics 11 Source Book.*

Turn to the table on page 106 of the textbook and take note of the skills and concepts listed for each of the tutorials; then read the Important Results and Formulas column. The table indicates concisely what you should know and should be able to do after participating in the learning activities of this module.

The following exercise provides an opportunity for you to apply several concepts and skills at the same time.



Turn to page 107 of the textbook and do exercise 1 of Part A of "What Should I Be Able To Do?"

Compare your responses with the suggested answers in the Appendix, Follow-up Activities, pages 127–130.

If you had difficulties understanding the skills and concepts in Module 2: Non-Linear Functions, it is recommended that you do the Extra Help. If you have a clear understanding of the skills and concepts in this module, it is recommended that you do the Enrichment. You may decide to do both.

EXTRA HELP

In Activity 4 you transformed linear equations into functions to find the zeros using a graphing calculator. You may also prefer to use this method for non-linear equations.

Step 1: Rearrange the equation so there is a zero on one side and a simplified algebraic expression on the other.

Step 2: Make a function by setting $f(x)$ equal to the algebraic expression. Remember: $f(x)$ and y are the same.

Step 3: Find the zeros of the function, which yield the solution to the original equation.

Example

Rearrange the equation $x^2 + 6x = x - 25$ so you get zero on one side and a simplified expression on the other. Then indicate the function whose zeros yield the solution to $x^2 + 6x = x - 25$.

Solution

$$x^2 + 6x = x - 25$$

$$x^2 + 6x - x = x - 25 - x \quad \longleftarrow \text{Subtract } x \text{ from both sides.}$$

$$x^2 + 5x = -25$$

$$x^2 + 5x + 25 = -25 + 25 \quad \longleftarrow \text{Add 25 to both sides.}$$

$$x^2 + 5x + 25 = 0$$

$$\therefore f(x) = x^2 + 5x + 25 \quad \longleftarrow \text{Set } f(x) \text{ equal to the algebraic side.}$$

The function is $f(x) = x^2 + 5x + 25$.

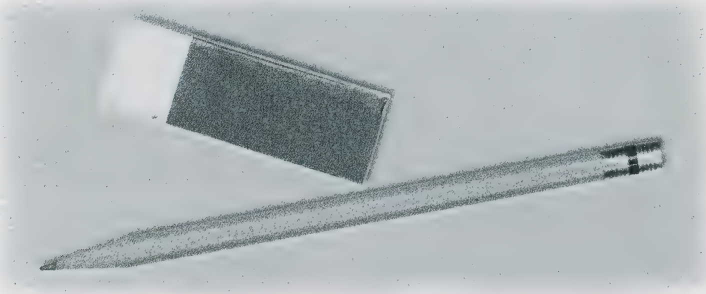
1. Rearrange each of the following equations and indicate the function whose zeros are solutions to the given equation.

   a. $9x^3 - 5x^4 = 20x^3 + 10$

   b. $4x^3 - 66 = 20x^2 + 200x$

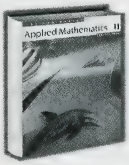
2. Determine the solution to $2x^3 + x^2 + 7 = 8x^2 + 2x$ using the method of transforming the non-linear equations into a function and using a graphing calculator to find the zeros.

Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Extra Help, pages 131–132.



ENRICHMENT

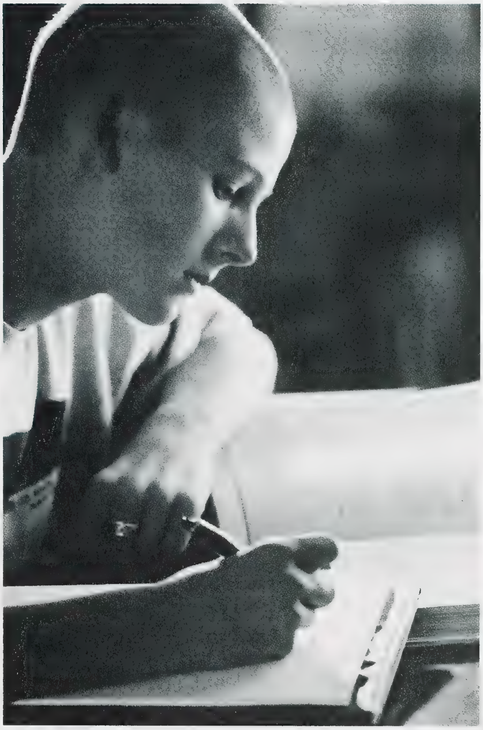
You have used your graphing calculator to determine characteristics of graphs of quadratic functions. Did you know that the factorized form of a quadratic function can reveal some of these characteristics directly?



1. Turn to page 81 of the textbook and answer exercise 7 of "Exercises: Extending Your Thinking."

2. Without graphing, determine the zeros and the axis of symmetry for $y = x^2 + 3x + 2$.

Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Enrichment, pages 132–134.

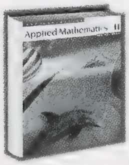


MODULE PROJECT

THE WORLD'S POPULATION

Completing the Project

You should now have done much of the research on world population growth and its relation to Earth's resources. But total world population trends may hide the disparity in the quality of life over different regions of the world.

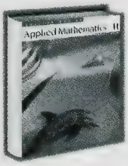


Turn to page 82 of the textbook and read the first paragraph of "Modelling the Growth of the World's Population."

1. Answer exercise 1 of "Modelling the Growth of the World's Population" on page 82 of the textbook. Use the type of graph that will best represent the data and explain why it best represents the data.

Compare your response with the suggested answer in the Appendix, Module Project, page 134.

The remaining exercises serve as a guide as you model population growth and make predictions based on your models. Your answers will help you in completing your project.



Turn to pages 104 and 105 of the textbook and read "A Logistic Model for the Growth of the World's Population." Enter the WORLDPOP program given on page 422 of the textbook. Instructions as to how to enter this program are given in the TI-83 Programs section of the Appendix of this Student Module Booklet.

If you are using a calculator other than the TI-83, the *Excel*® spreadsheet titled *POPULA.XLS* can be used in place of the WORLDPOP program. This program is located on the Applied Mathematics 20 CD. You can use this spreadsheet program to explore how spreadsheet technology can be used to predict future population growth. However, your answers will be slightly different in the first or second decimal place compared to those obtained with the WORLDPOP program on the TI-83.

Use the WORLDPOP program and follow the instructions on pages 104 and 105 of the textbook. Complete as much of exercises 1 and 2 on page 105 as you feel is necessary to make you familiar with using the WORLDPOP program. Compare the results you obtain with those given in the tables on page 109.

You may use the features from the CALCULATE menu in place of the Trace feature to determine the population for specific years. **Note:** The calculator may take several minutes to determine the equation using "B:Logistic" from the CALC menu.

Assessing a sample of student work can help you in the preparation of your own final submission. You may wish to review the expectations set for the project on page 62 of the textbook before doing the assessment in the next exercise.

2. Turn to pages 109 and 110 of the textbook and answer exercises 8 and 9 of Part C of "What Should I Be Able To Do?"

Compare your responses with the suggested answers in the Appendix, Module Project, page 135.

The Module Project

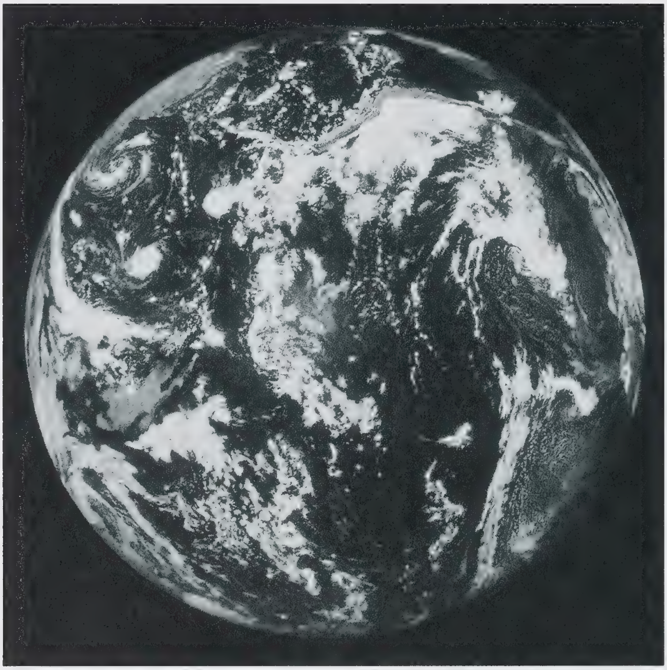
Now that you have more insight into what is required for the module project, turn to your Project Booklet and complete the module project, The World's Population.

Submit your completed Module 2 Project Booklet to your teacher.

Module Summary

In this module, you plotted data and determined exponential and quadratic regression functions for the data; determined key characteristics of the graphs of quadratic functions; solved problems using quadratic functions; and modelled data using regression functions. You also solved non-linear equations using a graphing tool.

So far, the human population has not reached a number that cannot be sustained by Earth's resources. Your skill in modelling the world's population was based on past trends, which helped in providing some insight into how Earth's population could continue to grow. The population growth that does occur will determine the demands on Earth's limited resources and the material well-being of Earth's inhabitants.



Module Assignment

To demonstrate what you have learned in this module, complete the module assignment in the Assignment Booklet.

Submit your completed Module 2 Assignment Booklet to your teacher.

APPENDIX

Glossary
Suggested Answers
T1-83 Programs
Image Credits

GLOSSARY

axis of symmetry: a line about which a geometric figure is symmetrical

decay function: an exponential function that decreases with an increase in the independent variable

exponential function: a function in which the independent variable is an exponent

exponential growth function: an exponential function that increases with an increase in the independent variable

half-life: the period of time in which the mass of an unstable substance diminishes by a factor of one-half

parabola: the name of the shape of the graph of a quadratic function

quadratic function: a function of the form

$$f(x) = ax^2 + bx + c$$

regression curve: a best-fit curve for a set of data points

system of equations: two or more equations that are considered at the same time

trajectory: the curved path taken by an object thrown or propelled through the air or space

vertex: the point corresponding to the minimum or maximum value of a quadratic function

zero of a function: the value of the independent variable for which the function has a value of 0

SUGGESTED ANSWERS

Activity 1: Exponential Graphs

1. Textbook exercises 1, 2.b., and 2.c. of "Practise Your Prior Skills," p. 64

1. a.
$$\begin{aligned}\left(x^3\right)^4 &= x^{3\times4}\\ &= x^{12}\end{aligned}$$

b.
$$\begin{aligned}3y^0 &= 3\times1\\ &= 3\end{aligned}$$

c.
$$\begin{aligned}\frac{\left(2x^5\right)\left(x^3\right)}{\left(6x^4\right)\left(x^7\right)} &= \frac{\overset{1}{\cancel{2}}x^{5+3}}{\underset{3}{\cancel{6}}x^{4+7}}\\ &= \frac{x^8}{3x^{11}}\\ &= \frac{1}{3x^{11-8}}\\ &= \frac{1}{3x^3}\end{aligned}$$

d.
$$3y^{-4} = \frac{3}{y^4}$$

2. b. $g(x)=\sqrt{x-1}$

The value under the square root sign must not be negative. Therefore, the domain is

$$\begin{aligned}x-1 &\geq 0\\ x &\geq 1\end{aligned}$$

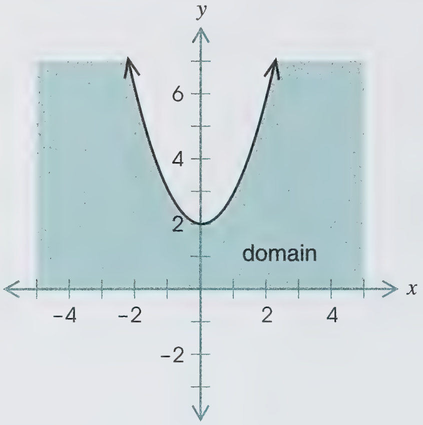
Because the value of the function is expressed with the square root sign $\left(\sqrt{\ }\right)$, the function is always positive.

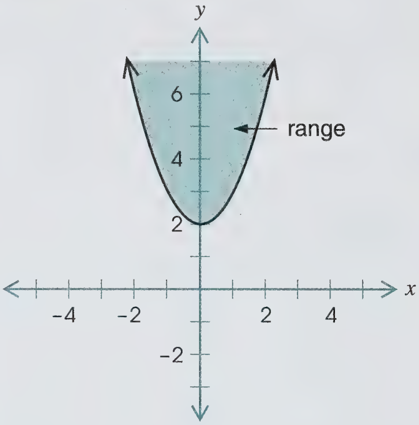
If x is 1, then

$$\begin{aligned}g(x) &= g(1)\\ &= \sqrt{1-1}\\ &= \sqrt{0}\\ &= 0\end{aligned}$$

For any other value of x greater than 1, $g(x)$ will be a number greater than 0. Therefore, the range is $g(x)\geq 0$.

c. Sketching the graph will help determine the domain and range of $h(x) = x^2 + 2$.

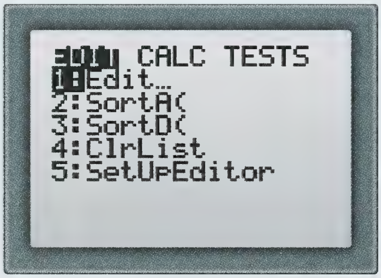


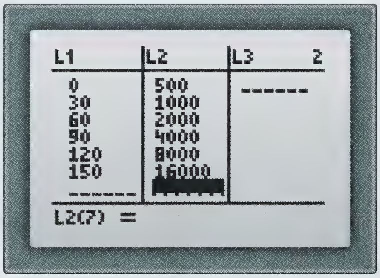


The domain is the set of real numbers, and the range is $h(x) \geq 2$.

2. Textbook exercises 1 to 4 of "Investigation 1: Insect Colonies," pp. 64 and 65

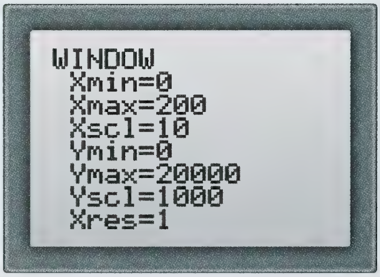
1. Press STAT 1, and enter the data.





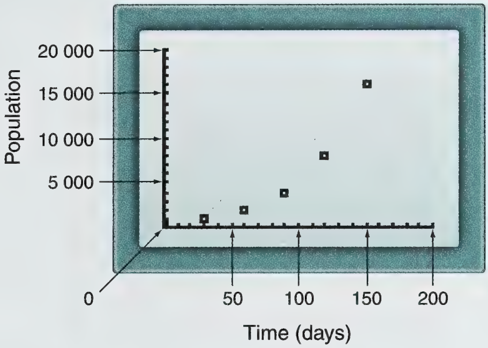
```
L1     |L2     |L3     2
0      |500    |------
30     |1000   |
60     |2000   |
90     |4000   |
120    |8000   |
150    |16000  |
------ |       |
L2(7) =
```

2. Step 1: Set appropriate window settings.

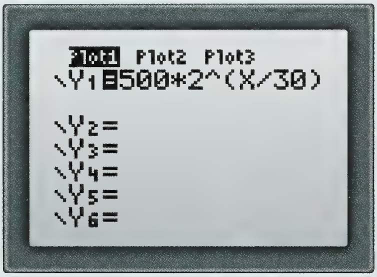


```
WINDOW
 Xmin=0
 Xmax=200
 Xscl=10
 Ymin=0
 Ymax=20000
 Yscl=1000
 Xres=1
```

Step 2: Press GRAPH to show the scatterplot.

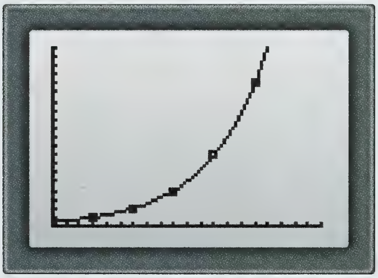


3. Y= 5 0 0 × 2 ^ (X,T,Θ,n ÷ 3 0) GRAPH



```
Plot1 Plot2 Plot3
\Y1■500*2^(X/30)

\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```



Note: Leave Plot 1 on so the scatterplot will also appear with the graph of the equation.

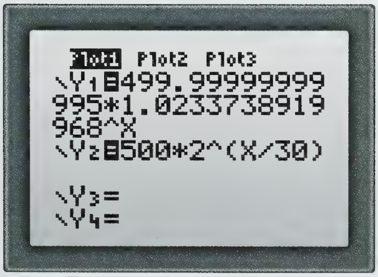
4. The equation that would model the growth is $y = 600 \times 2^{\frac{x}{30}}$.

3. **Textbook exercise 6 of "Investigation 1: Insect Colonies," p. 65**

6. **Step 1:** Enter $y = 500 \times 2^{\frac{x}{30}}$ as Y_2.

Place cursor beside Y_2.

Y= ▼ 5 0 0 × 2 ^ (X,T,Θ,n ÷ 3 0)

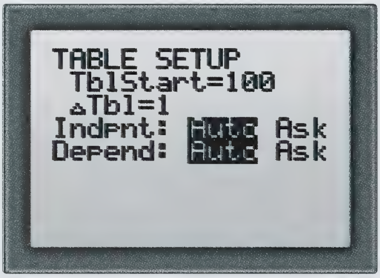


```
Plot1 Plot2 Plot3
\Y1■499.99999999
995*1.0233738919
968^X
\Y2■500*2^(X/30)

\Y3=
\Y4=
```

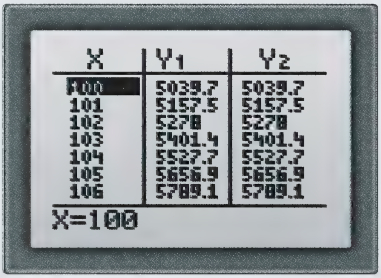
Activity 1 (continued)

Step 2: Press 2nd [TBLSET], and set the table to start at 100.



```
TABLE SETUP
 TblStart=100
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

Step 3: Press 2nd [TABLE] to display the table of values.



```
  X    | Y1     | Y2
100    | 5039.7 | 5039.7
101    | 5157.5 | 5157.5
102    | 5278   | 5278
103    | 5401.4 | 5401.4
104    | 5527.7 | 5527.7
105    | 5656.9 | 5656.9
106    | 5789.1 | 5789.1
X=100
```

Both equations predict the number of insects to be about 5040 after 100 d.

Note: You could have substituted 100 into each of the equations on your calculator and have seen that the results were very nearly equal.

Notice that the graphs of $y = 500 \times 2^{\frac{x}{30}}$ and $y = 500 \times 1.023\ 373\ 892^{x}$ fit the data. In fact, the graphs for these functions coincide. How can two different-looking functions be the same? The laws of exponents can show the equivalence of these functions.

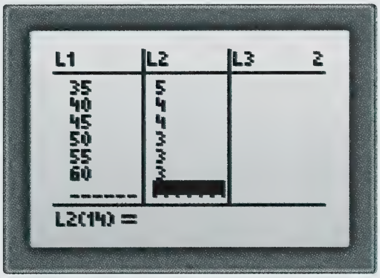
$$
\begin{aligned}
y &= 500 \times 2^{\frac{x}{30}} \\
&= 500 \times 2^{\left(\frac{1}{30} \times x\right)} \\
&= 500 \times \left(2^{\frac{1}{30}}\right)^{x} \\
&= 500 \times 1.023\ 373\ 892^{x}
\end{aligned}
$$

$a^{mn} = \left(a^{m}\right)^{n}$

4. Textbook exercise 5 of "Investigation 2: Cooling Temperatures," p. 67

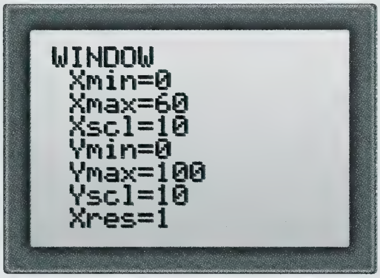
5. Answers may vary. This answer is based on the data that was given.

Step 1: Enter the data.

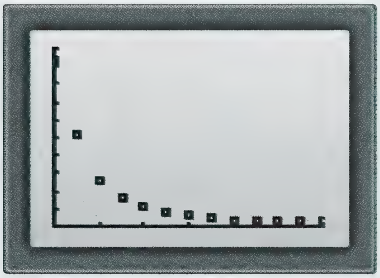


```
L1      |L2      |L3     2
  35    | 5      |
  40    | 4      |
  45    | 4      |
  50    | 3      |
  55    | 3      |
  60    | 3      |
 ------ |        |
L2(14) =
```

Step 2: Using appropriate windows settings, graph the data by pressing GRAPH. The graph shows the trend of the data.



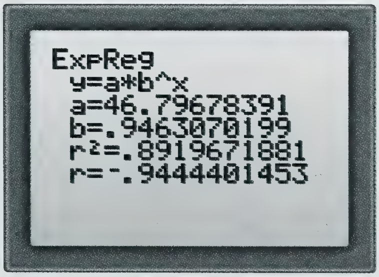
```
WINDOW
 Xmin=0
 Xmax=60
 Xscl=10
 Ymin=0
 Ymax=100
 Yscl=10
 Xres=1
```



Step 3: Determine the regression equation. Because the graph looks like an exponential curve, use the Exponential Regression feature.

Select the CALC menu.

STAT ▶ 0 (0:ExpReg) ENTER



```
ExpReg
 y=a*b^x
 a=46.79678391
 b=.9463070199
 r²=.8919671881
 r=-.9444401453
```

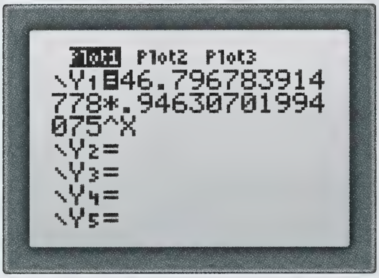
The value of r^2 is close to 1. This shows a fairly close, but not exact, fit of the curve to the data.

Activity 1 (continued)

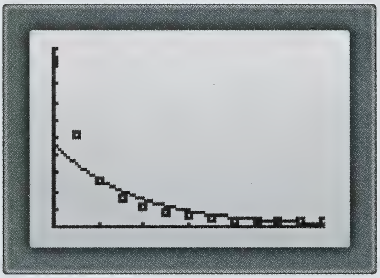
Step 4: Graph the regression equation to visually confirm the suitability of the equation.

Select the EQ menu.

Y= VARS (5:Statistics…) ▶ ▶ 1 (1:RegEQ) GRAPH



```
Plot1 Plot2 Plot3
\Y1■46.796783914
778*.94630701994
075^X
\Y2=
\Y3=
\Y4=
\Y5=
```



The regression equation is $y=(46.796\,783\,91)(0.946\,307\,019\,9)^x$. **Remember:** Your equation may be different.

5. **Textbook exercise 2 of "Discussing the Ideas," p. 70**

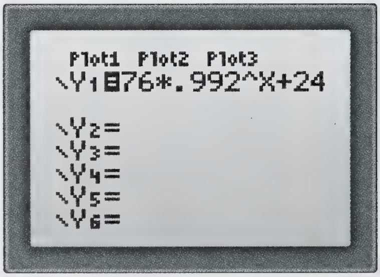
2. It is appropriate to join the points on the graph because there is a temperature for all the times between the times at which the temperatures were measured.

6. **Textbook exercise 8 of "Exercises: Extending Your Thinking," p. 74**

8. **a.**

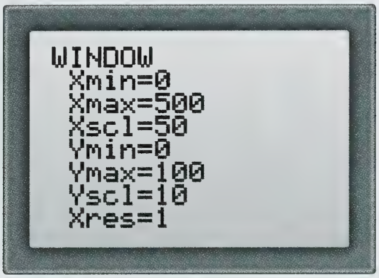
$$\begin{aligned} T &= A\left(B^t\right)+C \\ &= (100-24)\left(0.992^t\right)+24 \\ &= 76(0.992)^t+24 \end{aligned}$$

Step 1: Enter the equation into the equation editor.

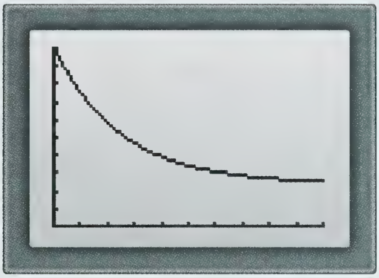


```
Plot1 Plot2 Plot3
\Y1■76*.992^X+24
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

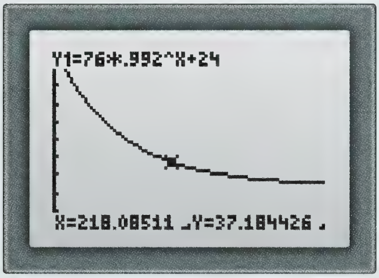
Step 2: Graph the equation by pressing GRAPH.



```
WINDOW
 Xmin=0
 Xmax=500
 Xscl=50
 Ymin=0
 Ymax=100
 Yscl=10
 Xres=1
```



Step 3: Use the Trace feature to determine when the temperature reaches 37°C.



```
Y1=76*.992^X+24
X=218.08511  Y=37.184426
```

It would take water approximately 220 s (or 3 min 40 s) to cool to body temperature (37°C).

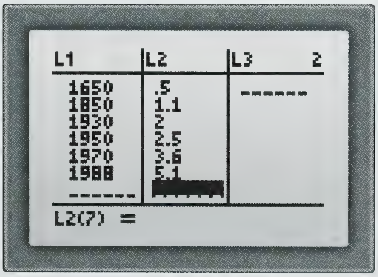
b. $T = A\left(B^t\right) + C$

$= (100 - 0)\left(0.9943^t\right) + 0$

The cooling equation is $T = 100\left(0.9943^t\right)$.

7. **Textbook exercises 2.a. to 2.c. of "Exercises: Checking Your Skills," p. 72**

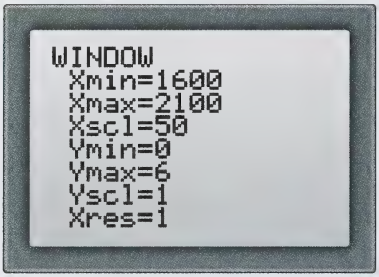
2. **a.** **Step 1:** Enter the data. For convenience, let the data in L2 represent billions.



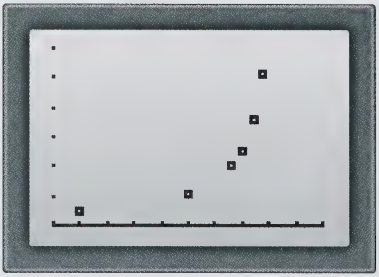
```
L1      |L2     |L3      2
  1650  | .5    |  ------
  1850  | 1.1   |
  1930  | 2     |
  1950  | 2.5   |
  1970  | 3.6   |
  1988  | 5.1   |
  ------|       |
L2(7) =
```

Activity 1 (continued)

Step 2: Using suitable window settings, graph the data.

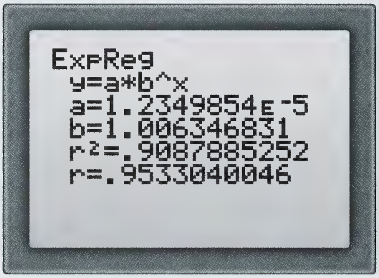


```
WINDOW
 Xmin=1600
 Xmax=2100
 Xscl=50
 Ymin=0
 Ymax=6
 Yscl=1
 Xres=1
```



b. STAT ▶ 0 ENTER

Select the CALC menu.



```
ExpReg
 y=a*b^x
 a=1.2349854E-5
 b=1.006346831
 r²=.9087885252
 r=.9533040046
```

“E-5” represents “$\times10^{-5}$.”

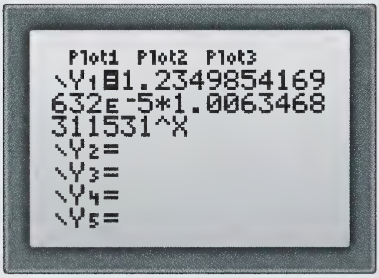
The exponential equation that best fits the data is $y=\left(1.234\,985\,4\times10^{-5}\right)\left(1.006\,346\,831\right)^{x}$.

c. Use the table of values to determine when the population reached 4 billion.

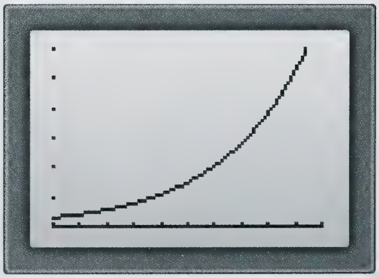
Step 1: Graph the regression equation. Make sure you turn Plot 1 off so the data points do not appear on the graph.

Select the EQ menu.

Y= VARS 5 ▶ ▶ 1 GRAPH

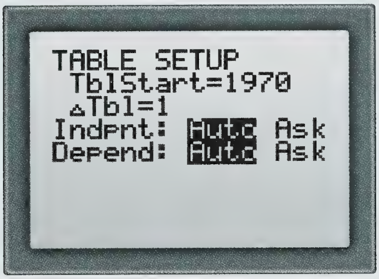


```
 Plot1 Plot2 Plot3
\Y1■1.2349854169
632E-5*1.0063468
311531^X
\Y2=
\Y3=
\Y4=
\Y5=
```



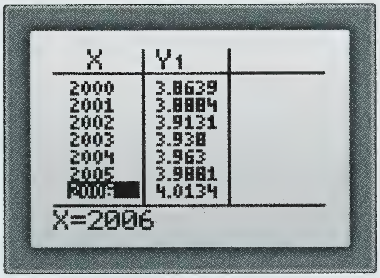
Step 2: Set up the table of values so it shows the population of each year starting with 1970.

Press 2nd [TBLSET], and set up the table of values as follows.



```
TABLE SETUP
 TblStart=1970
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

Step 3: Press 2nd [TABLE], and scroll down to the year where the projected population reaches 4 billion.



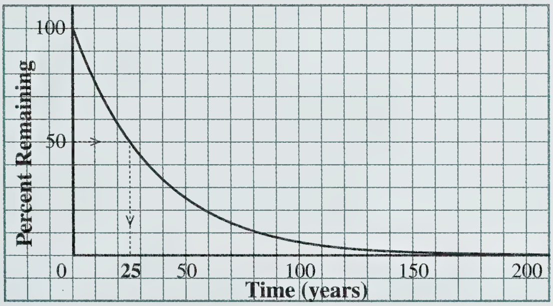
```
   X     Y1
  2000   3.8639
  2001   3.8884
  2002   3.9131
  2003   3.938
  2004   3.963
  2005   3.9881
  2006   4.0134
X=2006
```

According to the regression equation, the population will reach 4 billion in 2006.

According to the data given, the population exceeded 4 billion sometime between 1970 and 1988. This discrepancy shows that an equation that best fits a set of data must be used with caution.

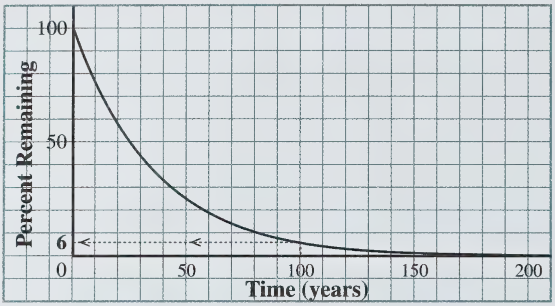
8. Textbook exercise 5 of "Exercises: Checking Your Skills," p. 73

5. a.



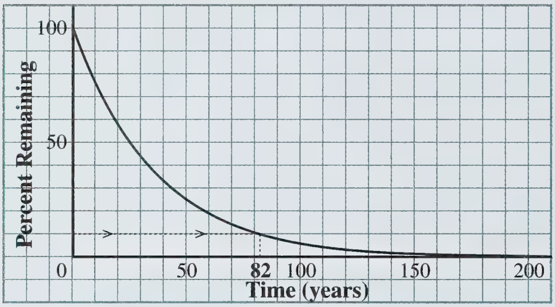
The half-life of strontium-90 is 25 years.

b.



After 100 years, only 6% of strontium-90 will remain.

c.



It will take approximately 82 years for strontium-90 to decay to 10% of its original mass.

9. Textbook exercises 4 and 5 of "Discussing the Ideas," p. 71

4. In a growth equation, the base is greater than 1. In a decay equation, the base is between 0 and 1 $(0 < \text{base} < 1)$.

5. In a growth equation, the base is greater than 1. In a decay equation, the base is between 0 and 1 $(0 < \text{base} < 1)$. (The answer is the same whether you use the calculator or use algebraic reasoning.)

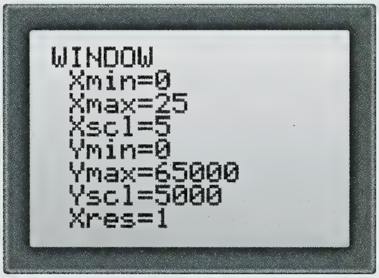
10. Textbook exercises 1, 3, and 4 of "Exercises: Checking Your Skills," pp. 71 and 72

1. a.

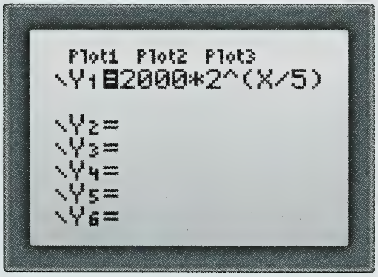
Time (h)	Population
0	2000
5	4000
10	8000
15	16 000
20	32 000
25	64 000

b. $y = 2000 \times 2^{\frac{t}{5}}$ or $y = 2000(1.148\,698\,355)^x$

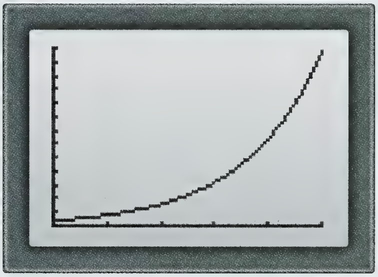
c. Using the window settings given, the graph of the growth curve looks like the following.



```
WINDOW
  Xmin=0
  Xmax=25
  Xscl=5
  Ymin=0
  Ymax=65000
  Yscl=5000
  Xres=1
```



```
  Plot1  Plot2  Plot3
\Y1■2000*2^(X/5)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

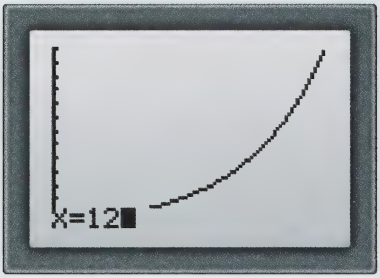


Activity 1 (continued)

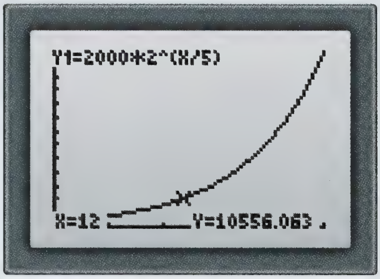
d. Determine the number of bacteria after 12 h and after 24 h using the Value feature from the CALCULATE menu.

Step 1: Find the number of bacteria after 12 h.

2nd [CALC] 1 (1:value) 1 2 ENTER



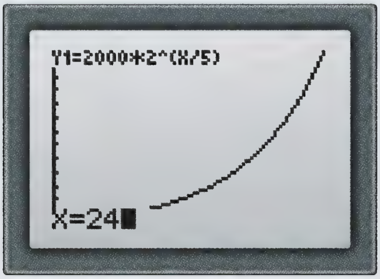
X=12



Y1=2000*2^(X/5)

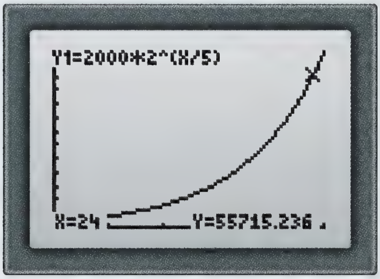
X=12 Y=10556.063

Step 2: Find the number of bacteria after 24 h. **Note:** At this point, you only have to enter the *x*-value. Therefore, simply press 2 4 ENTER.



Y1=2000*2^(X/5)

X=24



Y1=2000*2^(X/5)

X=24 Y=55715.236

There are approximately 10 556 bacteria after 12 h and approximately 55 715 bacteria after 24 h.

3. a.

Year	Population (millions)	Year	Population (millions)
1995	2.30	2003	3.03
1996	2.38	2004	3.13
1997	2.46	2005	3.24
1998	2.55	2006	3.36
1999	2.64	2007	3.48
2000	2.73	2008	3.60
2001	2.83	2009	3.72
2002	2.93	2010	3.85

b.

Year	Population (millions)
1995	2.30
1996	2.38
1997	2.46
1998	2.55

×1.035
×1.035
×1.035

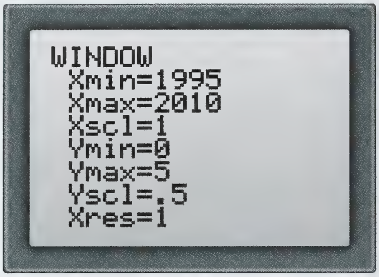
Using algebraic reasoning, the growth equation is $y = 2.3(1.035)^t$, where y is the population (in millions) and t is the number of years after 1995.

Instead of determining the number of years after 1995, transform the growth equation so that only the year needs to be known.

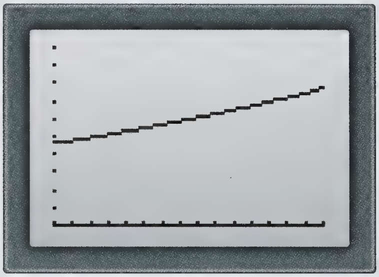
$$\begin{aligned} y &= 2.3(1.035)^t \\ &= 2.3(1.035)^{x-1995} \end{aligned}$$

Activity 1 (continued)

c. Using the given window settings, the graph of the growth should look like the following.

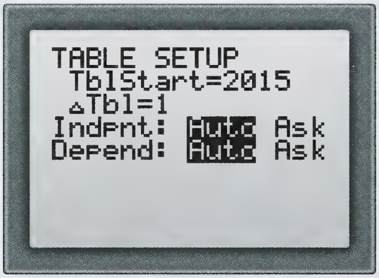


```
WINDOW
 Xmin=1995
 Xmax=2010
 Xscl=1
 Ymin=0
 Ymax=5
 Yscl=.5
 Xres=1
```

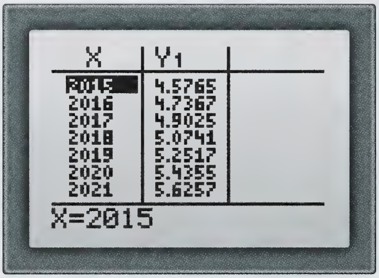


d. Use the table of values to determine the population in 2015.

Press 2nd [TBLSET], and set up the table so it starts at 2015; then press 2nd [TABLE], and read the population in 2015.



```
TABLE SETUP
 TblStart=2015
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```



X	Y₁
2015	4.5765
2016	4.7367
2017	4.9025
2018	5.0741
2019	5.2517
2020	5.4355
2021	5.6257

X=2015

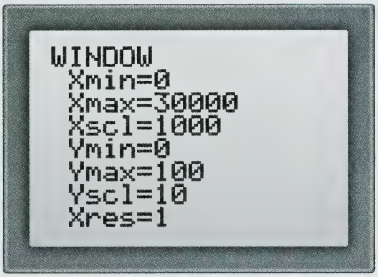
The population in 2015 will be approximately 4.58 million.

e. Using the table of values in the answer to exercise 3.d., the population will first exceed 5 million in 2018.

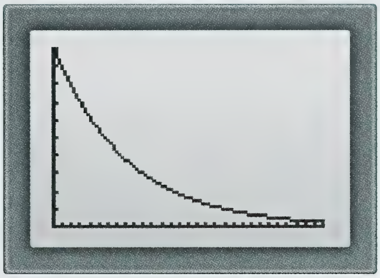
4. a. Algebraic Solution

$A = PM^{\frac{t}{T}}$	← This is the general equation for exponential decay (as given on page 70 of the textbook).
$\frac{A}{P} = M^{\frac{t}{T}}$	← Divide each side by P to give a ratio of the actual amount, A, of carbon-14 at a given time to the initial amount, P, of carbon-14.
$\frac{A}{P} = \left(\frac{1}{2}\right)^{\frac{t}{5700}} \times 100$	← $M = \frac{1}{2}$ and $T = 5700$. Because $\frac{A}{P}$ is a ratio, you need to multiply by 100 to change to percent.
$y = \left(\frac{1}{2}\right)^{\frac{x}{5700}} \times 100$	← Let y be the percent of carbon-14 remaining and x be the elapsed time (in years). This transforms the equation into a familiar function.
$y = 100 \times (0.5)^{\frac{x}{5700}}$	← Rearrange for convenience of entry into the calculator's equation editor.

b.

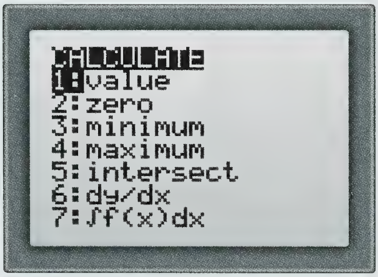


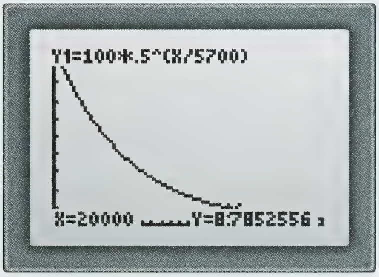
```
WINDOW
 Xmin=0
 Xmax=30000
 Xscl=1000
 Ymin=0
 Ymax=100
 Yscl=10
 Xres=1
```



c. Method 1: Using the Value Feature

2nd [CALC] 1 (1:value) 2 0 0 0 0 ENTER



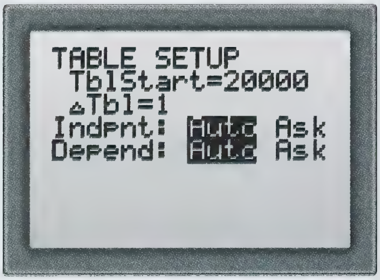


```
Y1=100*.5^(X/5700)
X=20000          Y=8.7852556
```

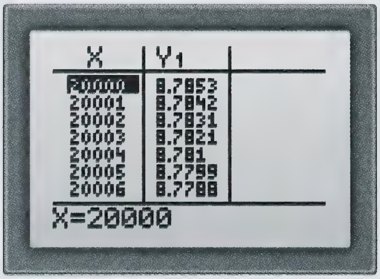
After 20 000 years, there is about 8.8% of carbon-14 remaining.

Method 2: Using the Table of Values

Press 2nd [TBLSET], and set up the table so it starts at 20 000; then press 2nd [TABLE], and read the percent of carbon-14 remaining after 20 000 years.



```
TABLE SETUP
 TblStart=20000
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

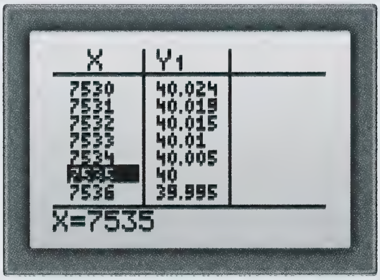


```
   X    | Y1     |
20000   | 8.7853 |
20001   | 8.7842 |
20002   | 8.7831 |
20003   | 8.7821 |
20004   | 8.781  |
20005   | 8.7799 |
20006   | 8.7788 |
X=20000
```

After 20 000 years, there is about 8.8% of carbon-14 remaining.

d. Method 1: Using the Table of Values

Estimate the value of x for $y = 40$ and set up the table of values so it starts at that point. You may need to estimate and set up the table more than once.

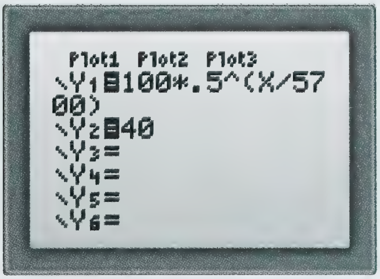


```
   X    | Y1     |
7530    | 40.024 |
7531    | 40.019 |
7532    | 40.015 |
7533    | 40.01  |
7534    | 40.005 |
7535    | 40     |
7536    | 39.995 |
X=7535
```

A sample having only 40% carbon-14 remaining is 7535 years old.

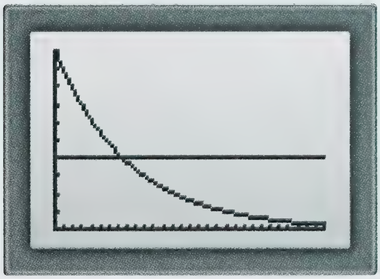
Method 2: Using the Intersect Feature

Step 1: Enter the function $y = 40$ as Y_2.



```
Plot1 Plot2 Plot3
\Y1=100*.5^(X/57
00)
\Y2=40
\Y3=
\Y4=
\Y5=
\Y6=
```

Step 2: Press GRAPH to graph both functions.



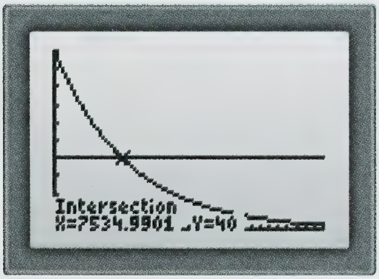
Step 3: Use the Intersect feature from the CALCULATE menu to find the point of intersection of the two graphs.

2nd [CALC] 5 (5:intersect) ENTER ENTER ENTER

chooses the first graph

chooses the second graph

determines the point of intersection



```
Intersection
X=7534.9901  Y=40
```

An object that shows 40% of carbon-14 remaining is approximately 7535 years old.

> Method 2 is much faster than Method 1. You solved a non-linear equation to arrive at the same answer (7535 years), which is the rounded *x*-value.

Activity 1 (continued)

11. Textbook exercise "Communicating the Ideas," p. 74

Both exponential growth equations and decay equations are of the form $y = ab^x$. For both equations, $b > 0$. However, $b > 1$ for exponential growth equations, whereas $b < 1$ for decay equations.

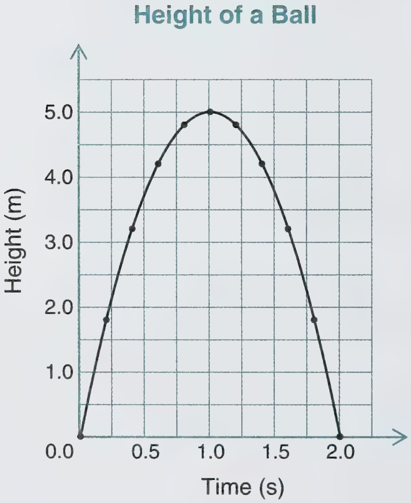
The equation $y = 100(1.05)^x$, where x is the time (in years) and y is the value of an investment (in dollars), corresponds to an investment of $100 experiencing an interest rate of 5%, compounded annually. The growth of the investment is exponential.

The equation $y = 100\left(\frac{1}{2}\right)^{\frac{x}{5700}}$, where x is the time (in years) and y is the percentage of carbon-14 remaining, corresponds to the decay of carbon-14, which has a half-life of 5700 years. This equation represents radioactive decay. Since $\left(\frac{1}{2}\right)^{\frac{1}{5700}} = 0.999\,878\,402\,6$, the decay can also be represented by $y = 100(0.999\,878\,402\,6)^x$. This equation is in the form $y = ab^x$. Note that although b is close to 1, it is slightly less than 1. Generally, the closer b is to 1, the longer the half-life of the substance undergoing exponential decay.

Activity 2: Quadratic Graphs

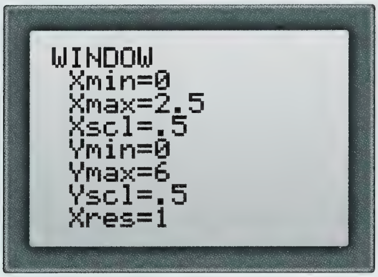
1. Textbook exercises 1 to 5 of Part B of "Investigation 1: What Goes Up," pp. 76 and 77

1. Method 1: Using Pencil and Paper

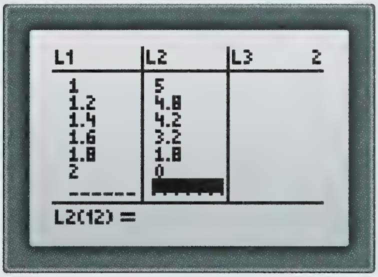


Method 2: Using a Graphing Calculator

Using the window settings given, press STAT 1 and enter the data.

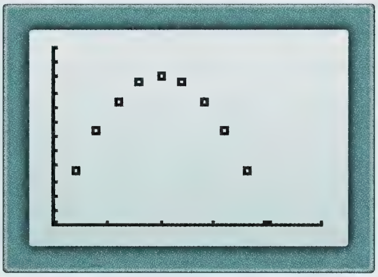


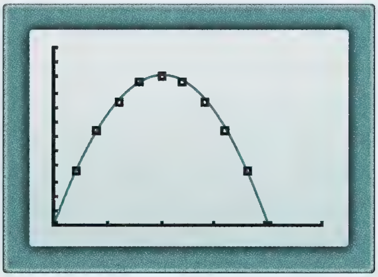
```
WINDOW
 Xmin=0
 Xmax=2.5
 Xscl=.5
 Ymin=0
 Ymax=6
 Yscl=.5
 Xres=1
```



```
L1      |L2      |L3      2
1        5
1.2      4.8
1.4      4.2
1.6      3.2
1.8      1.8
2        0
------
L2(12) =
```

Press GRAPH to view the points; then print the graph and draw a smooth curve through the points.

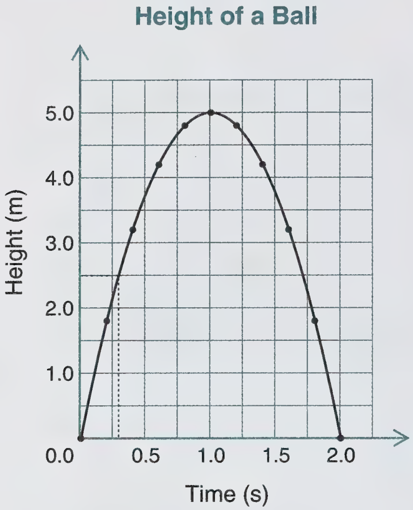


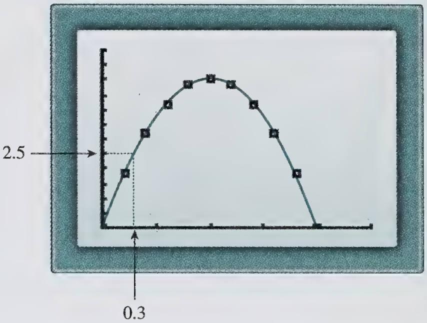


2. The coordinates of the maximum point is $(1.0, 5.0)$. This point shows that the ball reaches a maximum height of 5.0 m after 1.0 s.

Activity 2 (continued)

3.





It took the ball approximately 0.3 s to reach one-half its maximum height.

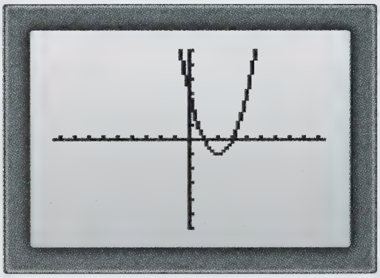
4. The x-intercepts occurs at $(0, 0)$ and $(2.0, 0)$. The first point shows the ball at the starting point, and the second point shows that the ball was in the air for 2.0 s.

5. The x-coordinate of the maximum point of the graph is half the total time the ball was in the air.

2. a. $a = 5, b = 2,$ and $c = 3$
   b. $a = 3, b = 0,$ and $c = -2$
   c. $a = 1, b = -1,$ and $c = 1$
   d. $a = 1, b = 0,$ and $c = 0$

3. **Textbook exercise 1 of "Investigation 2: The Characteristics of Quadratic Graphs," pp. 77 and 78**

1. a. Y= X,T,Θ,n x^2 − 4 X,T,Θ,n + 3 GRAPH

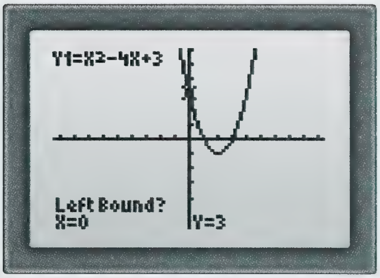


The graph is a parabola that opens upwards. The graph has a minimum point.

b. Find the first point where the graph crosses the *x*-axis.

Step 1: Press 2nd [CALC] 2 (2:zero) to activate the Zero feature.

The cursor appears at $(0, 3)$, and the calculator prompts you to choose the left bound.

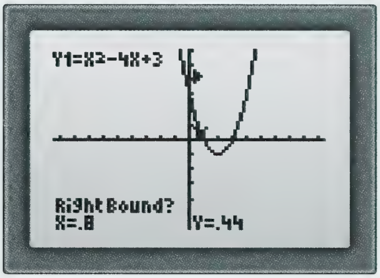


Y1=X²-4X+3

Left Bound?
X=0 Y=3

Step 2: Using the arrow keys, move the cursor so it appears just to the left of the zero. Then press ENTER to choose the left bound.

Now, the calculator prompts you to choose the right bound.

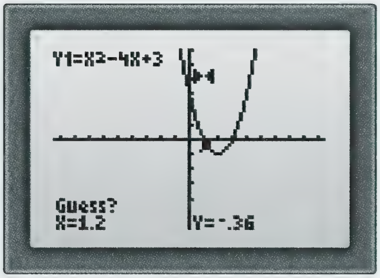


Y1=X²-4X+3

Right Bound?
X=.8 Y=.44

Step 3: Using the arrow keys, move the cursor so it appears just to the right of the zero. Then press ENTER to choose the right bound.

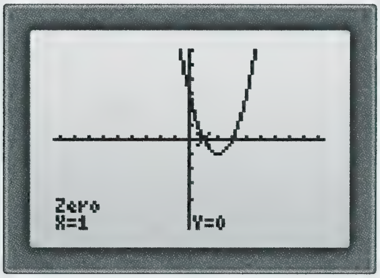
Now, the calculator prompts you to enter a guess.



Y1=X²-4X+3

Guess?
X=1.2 Y=-.36

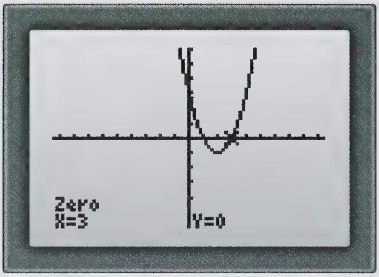
Step 4: Tap the left arrow key once and press ENTER to determine the first zero.



Zero
X=1 Y=0

The coordinates of the first point where the graph crosses the *x*-axis are $(1, 0)$.

Follow steps 1 to 4 to find the second point where the graph crosses the *x*-axis.

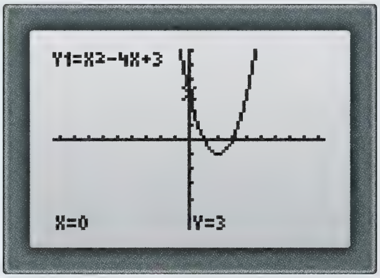


```
Zero
X=3             Y=0
```

The coordinates of the second point where the graph crosses the *x*-axis are $(3, 0)$.

c. Use the Value feature from the CALCULATE menu to determine where the graph crosses the *y*-axis.

2nd [CALC] 1 (1:value) 0 ENTER



```
Y1=X²-4X+3
X=0             Y=3
```

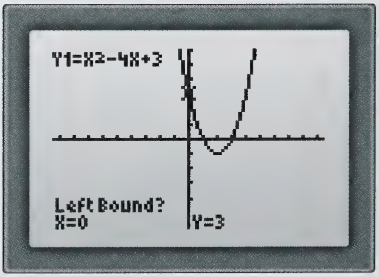
The coordinates of this point are $(0, 3)$. The *x*-coordinate is 0, and the *y*-coordinate is 3. The constant term of the equation is $+3$, which is the value of the *y*-intercept.

d. Use the Minimum feature from the CALCULATE menu to find the coordinates of the minimum point.

Step 1: Select the Minimum feature from the CALCULATE menu.

2nd [CALC] 3 (3:minimum)

The cursor appears at $(0, 3)$, and the calculator prompts you to choose the left bound.

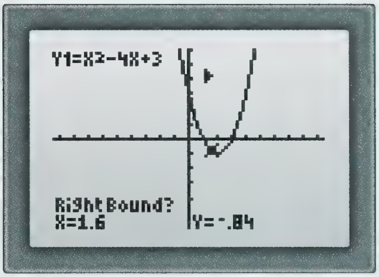


```
Y1=X²-4X+3
Left Bound?
X=0             Y=3
```

Step 2: Using the arrow keys, move the cursor just to the left of what appears to be the minimum value.

Then press ENTER.

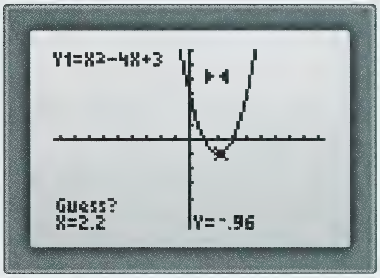
The calculator now prompts you to choose the right bound.



Step 3: Using the arrow keys, move the cursor just to the right of what appears to be the minimum value.

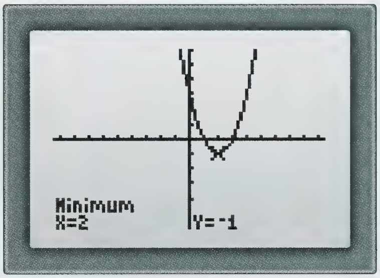
Then press ENTER.

The calculator now prompts you to enter a guess.



Step 4: Tap the left arrow key once and press ENTER to determine the minimum point.

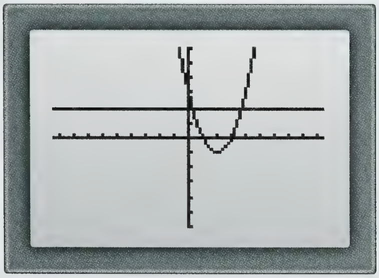
Note: The x-values may vary slightly depending on the left and right bounds chosen.



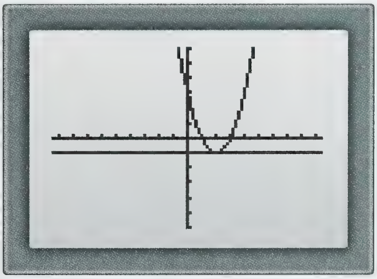
The minimum point is $(2,-1)$.

e. **Remember:** Clear the screen before activating the Draw feature. If the graph is showing when you activate the Draw feature, the horizontal or vertical line will appear wherever the cursor may be.

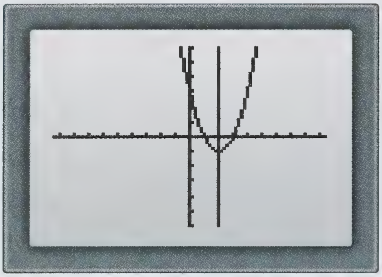
i. $y=2$



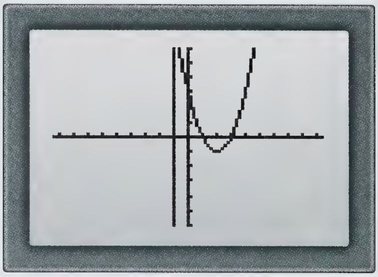
ii. $y=-1$



iii. $x = 2$



iv. $x = -1$



The line in part iii. is a line of symmetry. The equation of this line is $x = 2$.

f. The range is $y \geq -1$.

4. Textbook exercises 2.i. to 2.iv. of "Investigation 2: The Characteristics of Quadratic Graphs," p. 78

2.

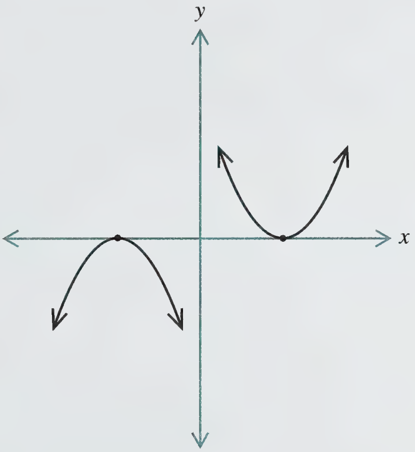
	$y=-x^2-2x+3$	$y=x^2-4x+7$	$y=5x^2-10x+5$	$y=-2x^2$
Shape	parabola opens downward	parabola opens upward	parabola opens upward	parabola opens downward
x-Intercept(s)	$(-3,0),(1,0)$	none	$(1,0)$	$(0,0)$
y-Intercept(s)	$y=3$ *	$y=7$ *	$y=5$ *	$y=0$ *
Minimum	NA	$(2,3)$	$(1,0)$	NA
Maximum	$(-1,4)$	NA	NA	$(0,0)$
Line of Symmetry	$x=-1$	$x=2$	$x=1$	$x=0$
Range	$y \leq 4$	$y \geq 3$	$y \geq 0$	$y \leq 0$

* The y-intercept is equal to the constant term of the equation.

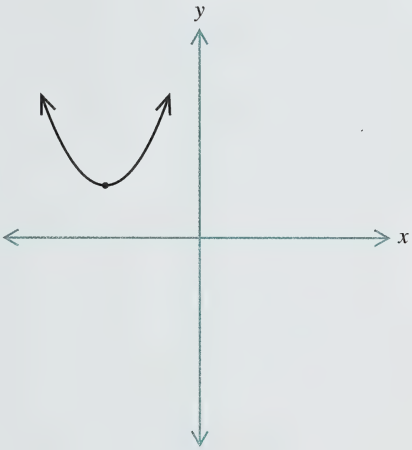
5. Textbook exercises 3 to 8 of "Discussing the Ideas," p. 79

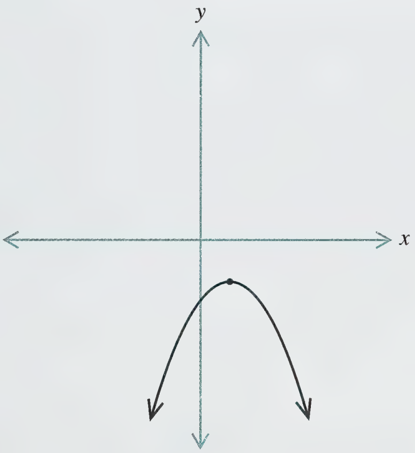
3. No, a quadratic function does not always have two zeros.

When the minimum or maximum of a graph of a quadratic function is on the x-axis, the quadratic function has exactly one x-intercept.



When the minimum of a graph is above the x-axis or the maximum of a graph is below the x-axis (as in exercise 2.ii.), the related function has no x-intercepts.





Activity 2 (continued)

4. In a quadratic function, if the coeffiecient of x^2 is positive, the graph has a minimum; if the coefficient of x^2 is negative, the graph has a maximum.

Coefficient of x^2	Maximum or Minimum?
Positive	Minimum
Negative	Maximum

5. The y-intercept corresponds to the constant term of the function.

6. The graph of a quadratic function will always have exactly one y-intercept because there is only one y-value for every x-value.

   A quadratic function is defined as having only one y-value for every x-value.

7. The domain of a quadratic function is always the set of real numbers because when any real number is substituted for x into the equation, a corresponding y-value can be determined.

8. For a parabola having a minimum point of $(3, -4)$, all y-values of the points of the parabola are greater than or equal to -4. Therefore, the range of the corresponding function is $y \geq -4$.

6. **A.** y-intercept
B. vertex
C. range
D. axis of symmetry
E. x-intercept

7. Textbook exercises 1.a., 2.a., 2.b., 3.a., 3.e., 6.a., 6.b., and 6.d. of "Exercises: Checking Your Skills," pp. 79 to 81

1. **a.** **i.** Yes. The function is of the form $y = ax^2 + bx + c$.

ii. Yes. The function can be rewritten as $y = -1x^2 + 0x + 25$, which is of the form $y = ax^2 + bx + c$.

iii. No. This function has an x^3-term.

iv. No. This function has terms under a square root sign.

v. No. This function has a term under a square root sign.

vi. No. This function has an x-term in the denominator.

2. a. i. The coordinates of the vertex are $(-1,-2)$.

ii. The equation of the axis of symmetry is $x=-1$.

iii. The x-intercepts are $(-2,0)$ and $(0,0)$. The y-intercept is $(0,0)$.

iv. The domain is all the real numbers. The range is $y \geq -2$.

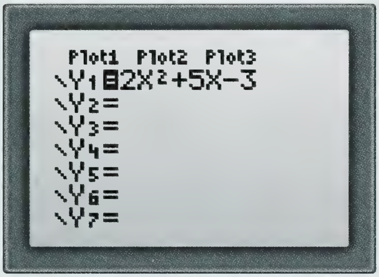
b. i. The coordinates of the vertex are $(3,4)$.

ii. The equation of the axis of symmetry is $x=3$.

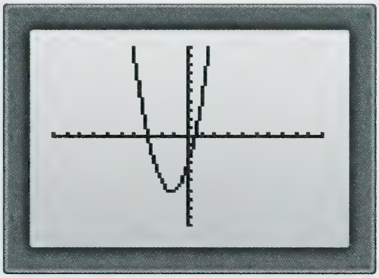
iii. The intercepts are $(1,0)$ and $(5,0)$. The y-intercept is $(0,-5)$.

vi. The domain is all the real numbers. The range is $y \leq 4$.

3. a. Using the standard window settings, by pressing ZOOM 6 (6:ZStandard), graph the equation.



```
Plot1 Plot2 Plot3
\Y1■2X²+5X-3
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

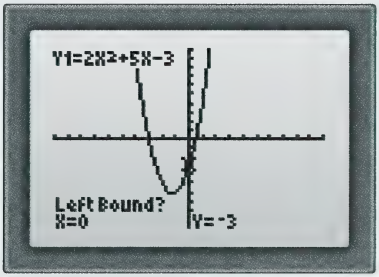


Because this graph has a minimum point, use the Minimum feature to find the equation of the axis of symmetry and the coordinates of the vertex.

Step 1: Select the Minimum feature from the CALCULATE menu.

2nd [CALC] 3 (3:minimum)

The cursor appears at $(0,-3)$, and the calculator prompts you to choose the left bound.

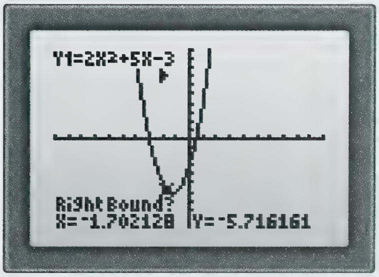


```
Y1=2X²+5X-3
Left Bound?
X=0          Y=-3
```

Step 2: Choose the left bound and the right bound.

Using the arrow keys, move the cursor just to the left of what appears to be the minimum point; then press ENTER.

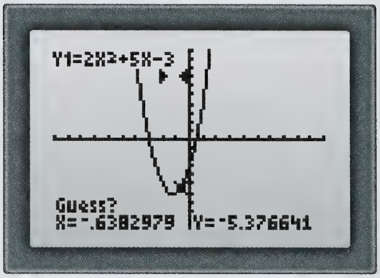
The calculator now prompts you to choose the right bound.



```
Y1=2X²+5X-3
Right Bound?
X=-1.702128  Y=-5.716161
```

Now, use the arrow keys to move the cursor just to the right of what appears to be the minimum point; then press ENTER.

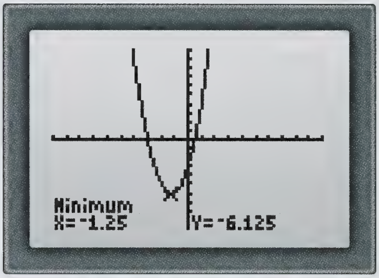
The calculator now prompts you to enter a guess.



$Y_1=2X^2+5X-3$

Guess?
X=⁻.6382979 Y=⁻5.376641

Step 3: Find the minimum point. Tap the left arrow key once and press ENTER.



Minimum
X=⁻1.25 Y=⁻6.125

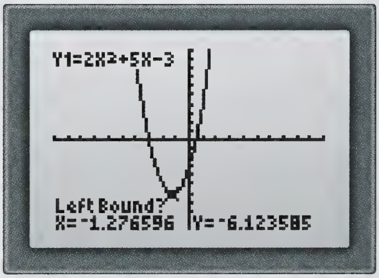
The minimum point occurs at $(-1.25, -6.125)$. Therefore, the equation of the axis of symmetry is $x=-1.25$, and the coordinates of the vertex are $(-1.25, -6.125)$.

Use the Zero feature to find the *x*-intercepts.

Step 1: Select the Zero feature from the CALCULATE menu.

2nd [CALC] 2 (2:zero)

The cursor appears near the last point referenced by the calculator, and the calculator prompts you to choose the left bound.



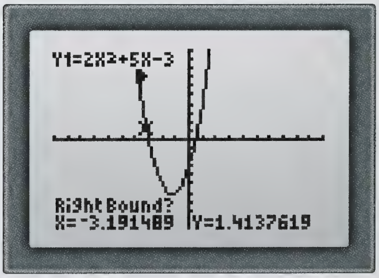
$Y_1=2X^2+5X-3$

Left Bound?
X=⁻1.276596 Y=⁻6.123585

Step 2: Choose the left bound and the right bound.

Using the arrow keys, move the cursor just to the left of what appears to an intercept; then press ENTER.

The calculator now prompts you to choose the right bound.

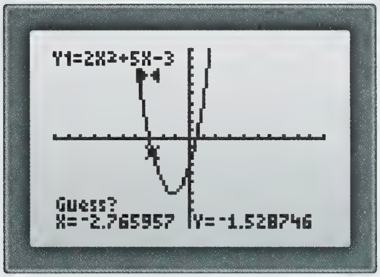


$Y_1=2X^2+5X-3$

Right Bound?
X=⁻3.191489 Y=1.4137619

Now, use the arrow keys to move the cursor just to the right of what appears to be an intercept; then press ENTER.

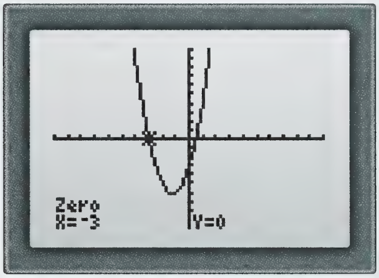
The calculator now prompts you to enter a guess.



```
Y1=2X²+5X-3
Guess?
X=⁻2.765957  Y=⁻1.528746
```

Step 3: Find the zero. Tap the left arrow key once and press ENTER.

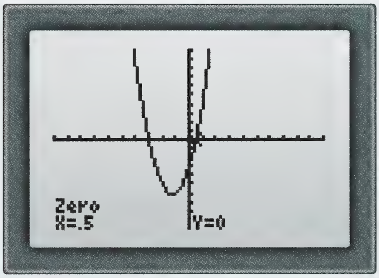
This zero is at $(-3, 0)$.



```
Zero
X=⁻3         Y=0
```

Step 4: Repeat steps 1 to 3 to find the other zero.

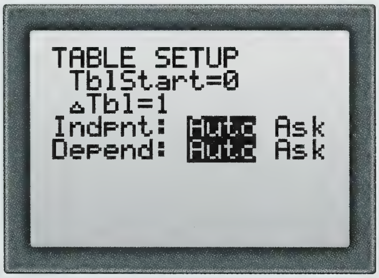
This zero is at $(0.5, 0)$.



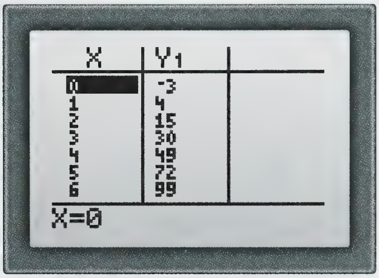
```
Zero
X=.5         Y=0
```

The x-intercepts are at $(-3, 0)$ and $(0.5, 0)$.

Use the table of values to find the y-intercept. Press 2nd [TBLSET] to set up the table so it starts at $x = 0$; then press 2nd [TABLE] to find the y-intercept.



```
TABLE SETUP
 TblStart=0
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

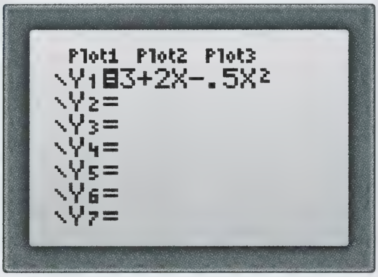


```
   X   | Y1  |
-------+-----+------
   0   | ⁻3  |
   1   | 4   |
   2   | 15  |
   3   | 30  |
   4   | 49  |
   5   | 72  |
   6   | 99  |
-------+-----+------
X=0
```

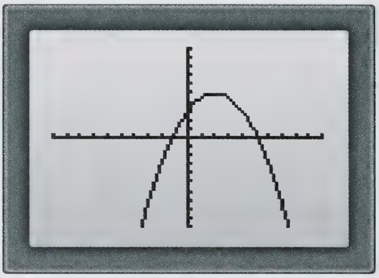
The y-intercept is at $(0, -3)$.

The domain is all the real numbers, and the range is $y \geq -6.125$. **Remember:** The minimum point shows the lowest possible y-value.

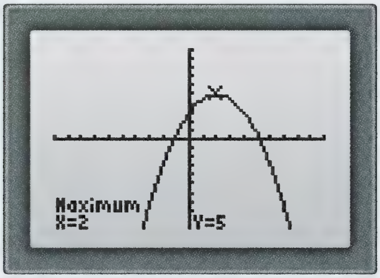
e. Graph the function. Use the same procedure as in the answer to exercise 3.a. to complete this exercise.



```
Plot1 Plot2 Plot3
\Y1■3+2X-.5X²
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



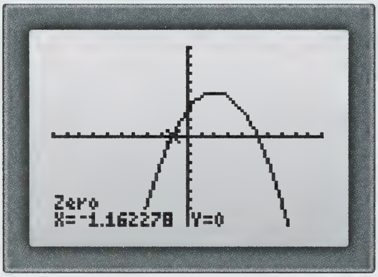
Because this graph has a maximum point, use the Maximum feature from the CALCULATE menu to find the equation of the axis of symmetry and the coordinates of the vertex.



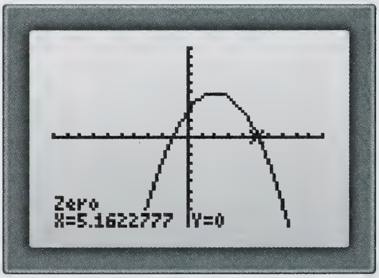
```
Maximum
X=2         Y=5
```

The equation of the axis of symmetry is $x = 2$, and the coordinates of the vertex are $(2, 5)$.

Use the Zero feature from the CALCULATE menu to find the *x*-intercepts.



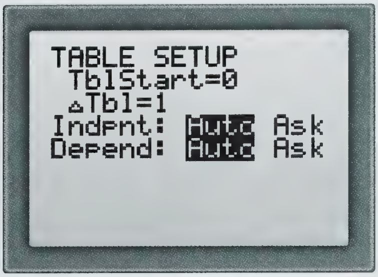
```
Zero
X=-1.162278   Y=0
```



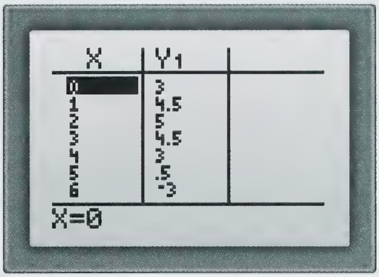
```
Zero
X=5.1622777   Y=0
```

The *x*-intercepts are approximately $(-1.16, 0)$ and $(5.16, 0)$.

Use the table of values to find the *y*-intercept.



```
TABLE SETUP
 TblStart=0
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```



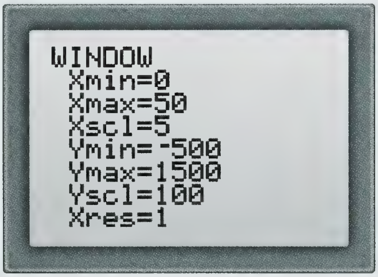
X	Y1
0	3
1	4.5
2	5
3	4.5
4	3
5	.5
6	-3

X=0

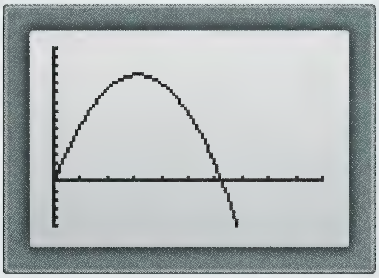
The *y*-intercepts is at $(0, 3)$.

The domain is all the real numbers, and the range is $y \le 5$.

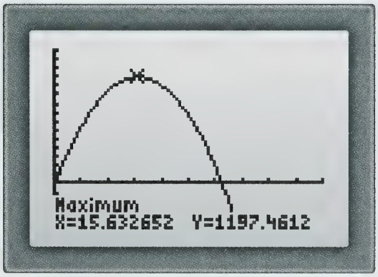
6. **a.** Using the window setting given, the graph of $h(t) = -4.9t^2 + 153.2t$ looks like the following.



```
WINDOW
 Xmin=0
 Xmax=50
 Xscl=5
 Ymin=-500
 Ymax=1500
 Yscl=100
 Xres=1
```



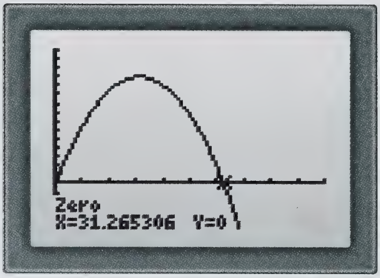
b. Use the Maximum feature from the CALCULATE menu to find the coordinates of the vertex.



Maximum
X=15.632652 Y=1197.4612

The coordinates of the vertex are approximately $(15.6, 1197.5)$.

Activity 2 (continued)



Zero
X=31.265306 Y=0

d. The function has input values only for the time the flare is in the air. This time period is represented by the spread between the zeros of the function. Use the Zero feature to determine when the flare hits the ground.

Therefore, the domain is approximately $0 \leq t \leq 31.27$.

Because height cannot be negative, the range is the values between ground level and the maximum height of the flare. Therefore, the range is approximately $0 \leq h(t) \leq 1197.5$.

8. Textbook exercise "Communicating the Ideas," p. 81

A **quadratic function** is a function of the form $y = ax^2 + bx + c$, where a cannot be zero; otherwise, the function would be linear. The graph of a quadratic function is a **parobola** that opens upwards if $a > 0$ and downwards if $a < 0$.

The **vertex** is the turning point of the parabola. If the parabola opens upwards $(a > 0)$, the vertex is the **minimum** point of the function. If the parabola opens downwards $(a < 0)$, the vertex is the **maximum** point of the function. The vertical line running through the vertex is called the **axis of symmetry**.

All quadratic functions have one **y-intercept**: $y = c$ or $(0, c)$, where c is the constant term of the function. The y-intercept is the point where the graph crosses the y-axis.

The **zeros of a quadratic function** are the x-values of the x-intercepts of the function. (The x-intercepts are the points $(x, 0)$ where the graph crosses the x-axis.) A quadratic function may have two, one, or no x-intercepts (as illustrated in the following chart).

Number of x-Intercepts	Maximum	Minimum
0	below the x-axis	above the x-axis
1	on the x-axis	on the x-axis
2	above the x-axis	below the x-axis

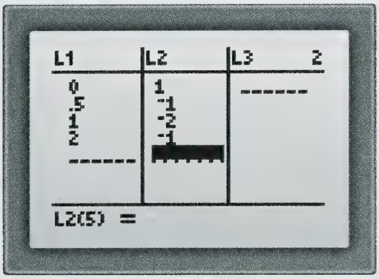
Activity 3: Problem Solving Using Quadratic Functions

1. **Textbook exercises 2 and 3 of "Exercises: Checking Your Skills," pp. 90 and 91**

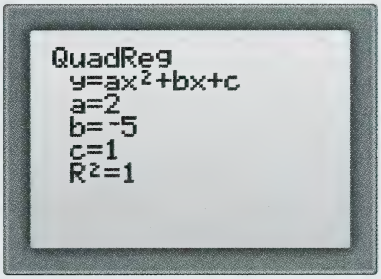
2. **a.** Press STAT 1 (1:Edit…) and enter the data. Then press the following to access the Quadratic Regression feature.

Select the CALC menu.

STAT ▶ 5 (5:QuadReg) ENTER



```
L1      |L2      |L3      2
 0      | 1      | ------
 .5     | ⁻1     |
 1      | ⁻2     |
 2      | ⁻1     |
 ------ |        |
L2(5) =
```



```
QuadReg
 y=ax²+bx+c
 a=2
 b=⁻5
 c=1
 R²=1
```

The quadratic regression equation is $y=2x^2-5x+1$. Because $a>0$, the graph has a minimum.

b. The quadratic regression equation is $y=-1x^2+20x$. Because $a<0$, the graph has a maximum.

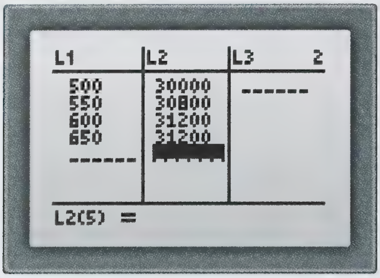
c. The quadratic regression equation is $y=1x^2-7x+10$. Because $a>0$, the graph has a minimum.

d. The quadratic regression equation is $y=-5x^2+100x$. Because $a<0$, the graph has a maximum.

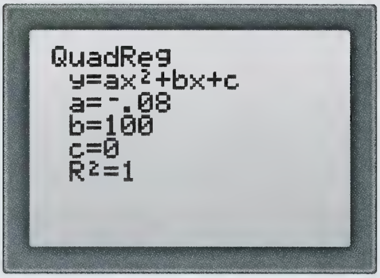
3. **a.**

Price per Canoe ($)	500	550	600	650
Canoes Sold	60	56	52	48
Total Revenue ($)	30 000	30 800	31 200	31 200

b.



```
L1      L2      L3     2
500     30000   ------
550     30800
600     31200
650     31200
------
L2(5) =
```



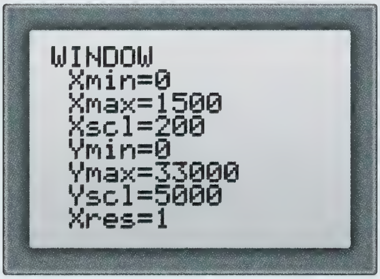
```
QuadReg
 y=ax²+bx+c
 a=-.08
 b=100
 c=0
 R²=1
```

The quadratic regression equation is $y = -0.08x^2 + 100x$.

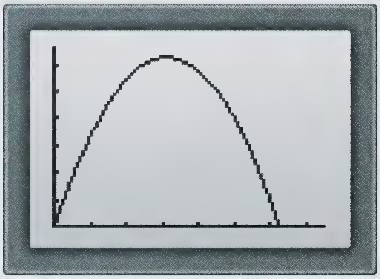
c. Using the given window settings, press the following keystrokes to graph the regression equation.

Select the EQ menu.

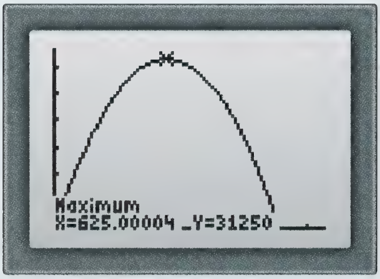
Y= VARS 5 (5:Statistics…) ▶ ▶ 1 (1:RegEQ) GRAPH



```
WINDOW
 Xmin=0
 Xmax=1500
 Xscl=200
 Ymin=0
 Ymax=33000
 Yscl=5000
 Xres=1
```



d. Use the Maximum feature from the CALCULATE menu.



```
Maximum
X=625.00004 Y=31250
```

The maximum is at $(625, 31\,250)$. Therefore, the optimum selling price is $625, which gives a maximum revenue of $31 250.

2. **Textbook exercises 2 and 3 of "Discussing the Ideas," p. 90**

   2. In the equation $h = -4.9t^2 + 8.8t + 3$, the last term corresponds to the 3-m springboard and the coefficient of the middle term corresponds to an initial velocity of $+8.8$ m/s.

   If Annie had dived off a 5-m springboard instead of a 3-m springboard and if the initial velocity is 7.2 m/s instead of 8.8 m/s, the equation becomes $h = -4.9t^2 + 7.2t + 5$.

   3. Negative values of y correspond to distances below the surface of the water.

   The equation does not hold for these negative values because the motion underwater is different from the falling motion above water. Once below the water surface, buoyancy and water resistance affect the diver's motion.

3. **Textbook exercises 1 and 7 of "Exercises: Checking Your Skills," p. 90 and 93**

   1. a.
$$\begin{aligned} h &= -\frac{1}{2}gt^2 + vt + s \\ &= -\frac{1}{2}(9.8)t^2 + 20t + 50 && \longleftarrow \text{Substitute } g = 9.8,\ v = 20, \text{ and } s = 50. \\ &= -4.9t^2 + 20t + 50 && \longleftarrow \text{Simplify.} \end{aligned}$$

   b.
$$\begin{aligned} h &= -\frac{1}{2}gt^2 + vt + s \\ &= -4.9t^2 + 10t + 0 \\ &= -4.9t^2 + 10t && \longleftarrow \text{Simplify.} \end{aligned}$$

   c.
$$\begin{aligned} h &= -\frac{1}{2}gt^2 + vt + s \\ &= -4.9t^2 + 0t + 3000 \\ &= -4.9t^2 + 3000 && \longleftarrow \text{Simplify.} \end{aligned}$$

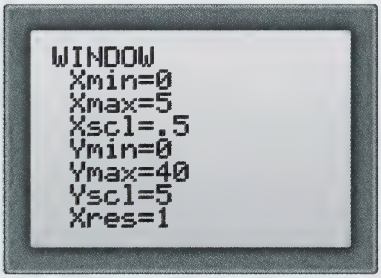
   d.
$$\begin{aligned} h &= -\frac{1}{2}g_{moon}t^2 + vt + s \\ &= -\frac{1}{2}(1.63)t^2 + 50t + 0 && \longleftarrow \text{Substitute } g = 1.63,\ v = 50, \text{ and } s = 0. \\ &= -0.815t^2 + 50t && \longleftarrow \text{Simplify.} \end{aligned}$$

7. **a.** $h = -\frac{1}{2}gt^2 + vt + s$

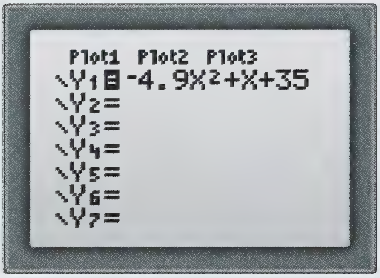
$= -\frac{1}{2}(9.8)t^2 + 1t + 35$ ← Substitute $g = 9.8$, $v = 1$, and $s = 35$.

$= -4.9t^2 + t + 35$ ← Simplify.

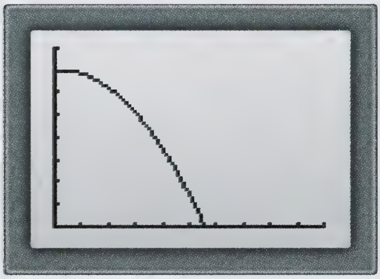
b. Using the given window settings, the graph should look like the following.



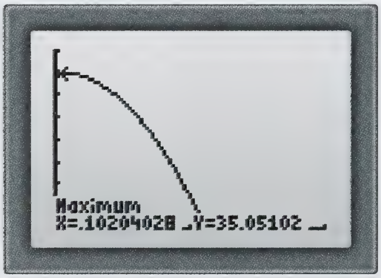
```
WINDOW
 Xmin=0
 Xmax=5
 Xscl=.5
 Ymin=0
 Ymax=40
 Yscl=5
 Xres=1
```



```
  Plot1 Plot2 Plot3
\Y1■-4.9X²+X+35
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



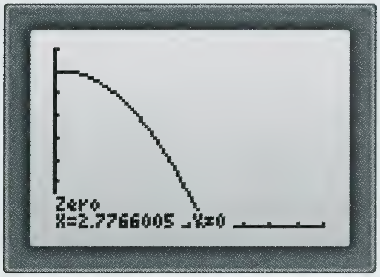
c. Use the Maximum feature from the CALCULATE menu.



```
Maximum
X=.10204028  Y=35.05102
```

The maximum height is about 35.05 m.

d. Use the Zero feature from the CALCULATE menu to find how long it takes the diver to hit the water.



```
Zero
X=2.7766005 Y=0
```

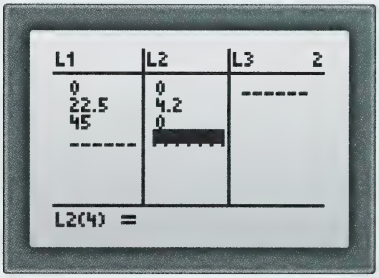
The diver is in the air for about 2.78 s.

4. Textbook exercises 9 and 10 of "Exercises: Checking Your Skills, " pp. 93 and 94

9. a. Let $(0, 0)$ be the point from where the ball is kicked.

The ball reaches its maximum height of 4.2 yards halfway along the ground to the end of the 45-yard kick. Therefore, $(22.5, 4.2)$ is the maximum point.

The ball hits the ground at $(45, 0)$.

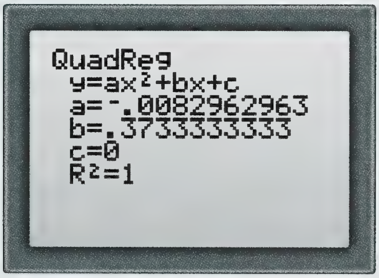


L1	L2	L3 2
0	0	------
22.5	4.2	
45	0	

L2(4) =

b. STAT ▶ 5 ENTER

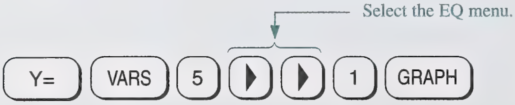
Select the CALC menu.



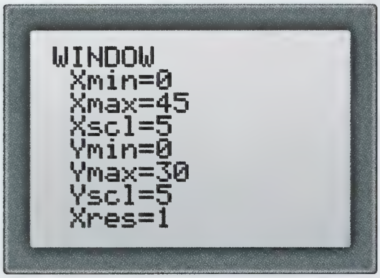
```
QuadReg
 y=ax²+bx+c
 a=-.0082962963
 b=.3733333333
 c=0
 R²=1
```

The equation is $y=-0.0083x^2+0.37x$.

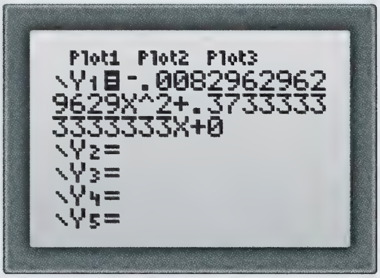
c. Using the window settings given, place the equation into the equation editor and graph the function.



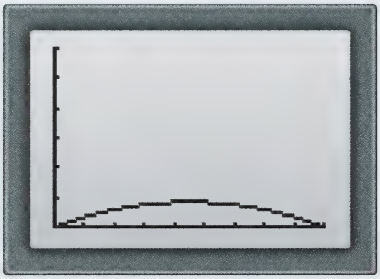
Select the EQ menu.



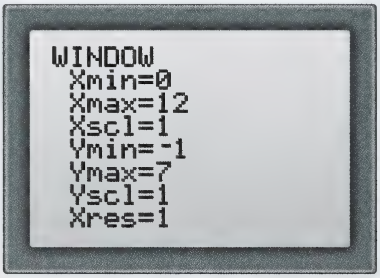
```
WINDOW
 Xmin=0
 Xmax=45
 Xscl=5
 Ymin=0
 Ymax=30
 Yscl=5
 Xres=1
```



```
  Plot1  Plot2  Plot3
\Y1■-.0082962962
9629X^2+.3733333
3333333X+0
\Y2=
\Y3=
\Y4=
\Y5=
```

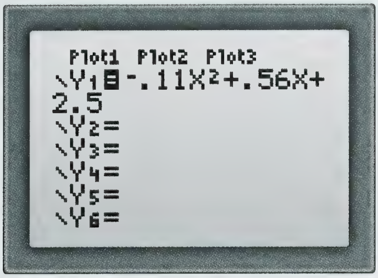


10. a. Using the given window settings, the graph of $h = -0.11d^2 + 0.56d + 2.5$ looks like the following.

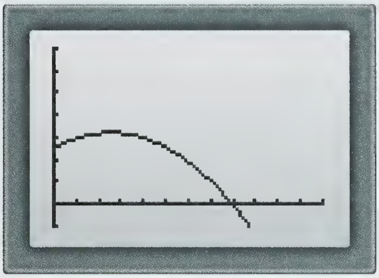


```
WINDOW
 Xmin=0
 Xmax=12
 Xscl=1
 Ymin=-1
 Ymax=7
 Yscl=1
 Xres=1
```

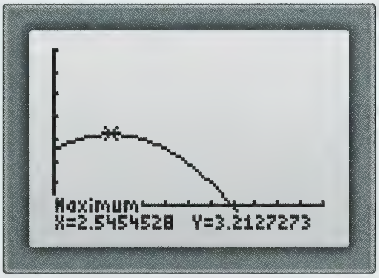
Ymax – Ymin is 8. This is $\frac{2}{3}$ of 12 (Xmax – Xmin). This ratio maintains the shape of the trajectory. You may have used other values that give the $\frac{2}{3}$ ratio.



```
Plot1 Plot2 Plot3
\Y1=-.11X²+.56X+
2.5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```



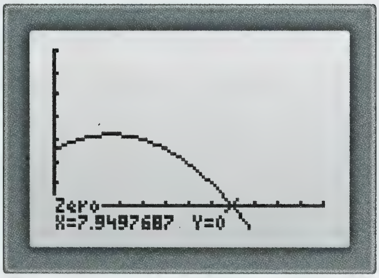
b. Use the Maximum feature from the CALCULATE menu.



```
Maximum
X=2.5454528  Y=3.2127273
```

The maximum height reached is about 3.21 m.

c. Use the Zero feature from the CALCULATE menu.



```
Zero
X=7.9497687  Y=0
```

The horizontal distance of the jump is about 7.95 m.

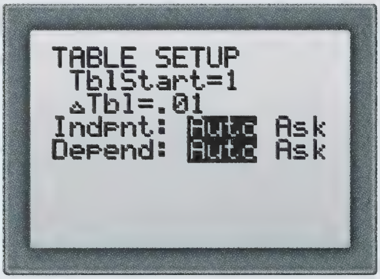
5. Textbook exercise 12 of “Exercises: Extending Your Thinking,” p. 94

12. Method 1: Using the Table of Values

The equation describing a stone dropping down a well is $h = -4.9t^2$. There are no other terms because $v = 0$ and $s = 0$.

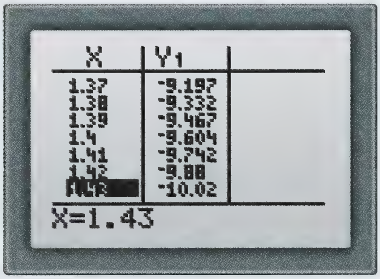
Step 1: Enter the function $y = -4.9t^2$ into the equation editor.

Step 2: Press 2nd [TBLSET] and set up the table like the one that follows.



```
TABLE SETUP
 TblStart=1
 ΔTbl=.01
Indpnt: Auto Ask
Depend: Auto Ask
```

Step 3: Press 2nd [TABLE] and scroll down to the y-value closest to -10.

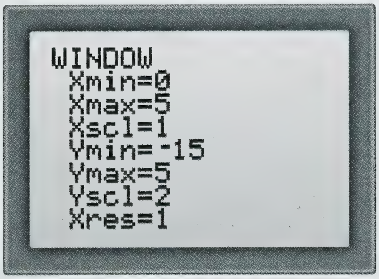


```
   X    | Y1     |
 1.37   | -9.197 |
 1.38   | -9.332 |
 1.39   | -9.467 |
 1.4    | -9.604 |
 1.41   | -9.742 |
 1.42   | -9.88  |
 1.43   | -10.02 |
X=1.43
```

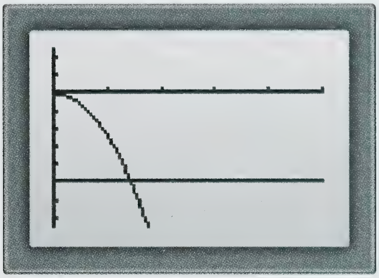
It takes approximately 1.43 s for the stone to fall the 10 m.

Method 2: Using the Intersect Feature

Using the window settings given, graph $y = -4.9t^2$ and $y = -10$ in the same window. Enter $y = -4.9t^2$ as Y_1 and $y = -10$ as Y_2.



```
WINDOW
 Xmin=0
 Xmax=5
 Xscl=1
 Ymin=⁻15
 Ymax=5
 Yscl=2
 Xres=1
```

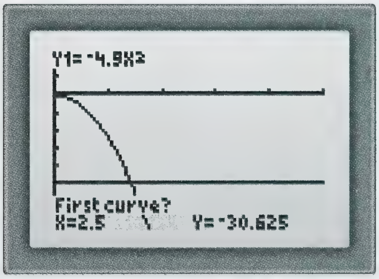


Find the intersection of the two graphs.

Step 1: Select the Intersect feature.

2nd [CALC] 5 (5:intersect)

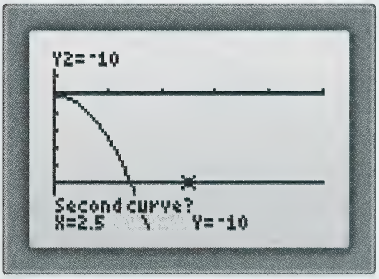
The cursor appears at $(0, 4)$, and the calculator asks if you wish to choose the graph of $y = -4.9x^2$ as the first curve.



```
Y1=⁻4.9X²
First curve?
X=2.5          Y=⁻30.625
```

Step 2: Press ENTER to choose the first curve.

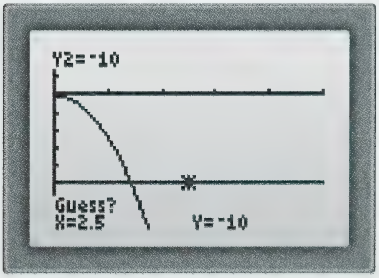
The calculator now asks if you wish to choose the graph of $y = -10$ as the second curve.



```
Y2=⁻10
Second curve?
X=2.5          Y=⁻10
```

Step 3: Press ENTER to choose the second curve.

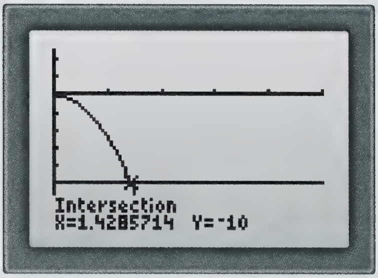
The calculator now asks you to enter a guess.



```
Y2=⁻10
Guess?
X=2.5          Y=⁻10
```

Activity 3 (continued)

Step 4: Using the arrow keys, move the cursor so it is close to the point of intersection; then press ENTER to find the point of intersection.



```
Intersection
X=1.4285714  Y=-10
```

It takes approximately 1.43 s for the stone to fall the 10 m.

> Method 2 is more accurate than Method 1. In Method 2 you used your calculator to solve a non-linear equation. Solving such equations is the topic of the next activity.

6. Textbook exercise "Communicating the Ideas," p. 94

A parabola has a turning point at which the curve changes direction—going from up to down or from down to up. This turning point is called the vertex of the parabola.

When the parabola opens downward, the vertex is the maximum point and potentially a very important point. For example, a maximum can indicate the maximum height of a trajectory or the optimum pricing of an item that will achieve maximum total revenue.

When the parabola opens upward, the vertex is the minimum point. The minimum point can also be important. The minimum point can indicate the lowest point in the swing of a pendulum or the lowest point in the dive of an airplane.

Activity 4: Solving Non-Linear Equations

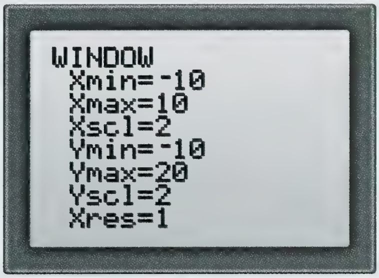
1.

LS	RS
$5x+4$	14
$=5(2)+4$	
$=14$	

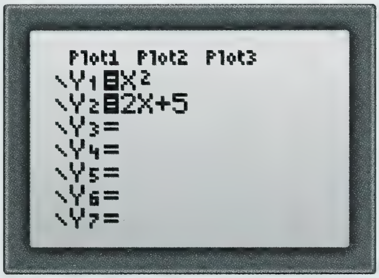
LS = RS

2. Textbook exercises 1 to 3 of "Investigation 1: Intersecting Graphs," p. 95

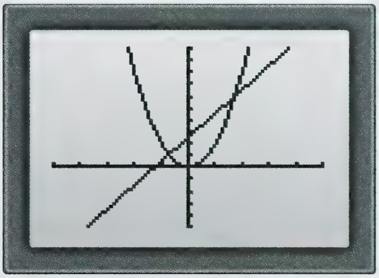
1. Using the given window settings, the graphs look like the following.



```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=2
 Ymin=-10
 Ymax=20
 Yscl=2
 Xres=1
```



```
  Plot1  Plot2  Plot3
\Y1=X²
\Y2=2X+5
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

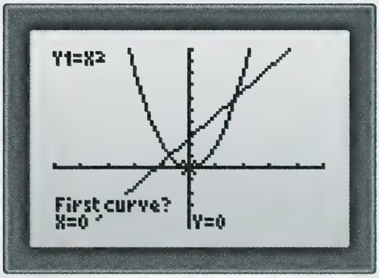


2 and 3. Use the Intersect feature on your graphing calculator to find the intersection points.

Step 1: Select the Intersect feature from the CALCULATE menu.

2nd [CALC] 5 (5:intersect)

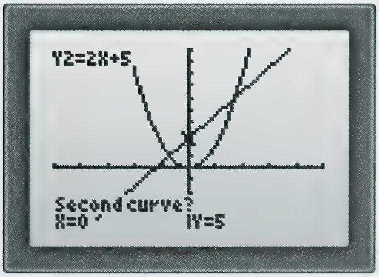
The cursor appears at $(0, 0)$, and the calculator asks if you wish to choose the graph of $y = x^2$ as the first curve.



```
Y1=X²
First curve?
X=0          Y=0
```

Step 2: Press ENTER to choose the first curve.

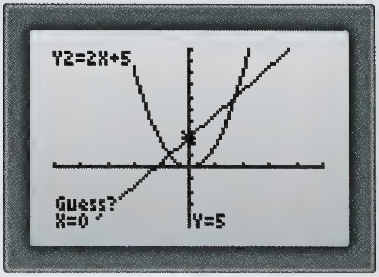
The calculator now asks if you wish to choose the graph of $y = 2x + 5$ as the second curve.



```
Y2=2X+5
Second curve?
X=0          Y=5
```

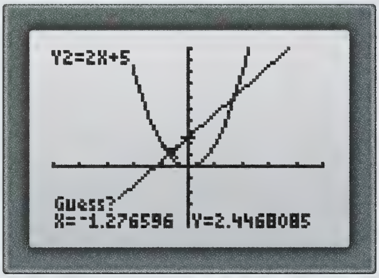
Step 3: Press ENTER to choose the second curve.

The calculator now asks you to enter a guess.

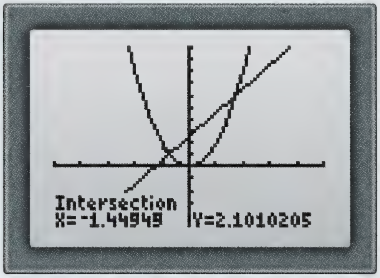


```
Y2=2X+5
Guess?
X=0          Y=5
```

Step 4: Using the arrow keys, move the cursor so it is close to the point of intersection; then press ENTER to find the point of intersection.

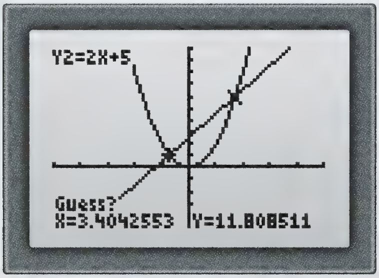


```
Y2=2X+5
Guess?
X=⁻1.276596  Y=2.4468085
```

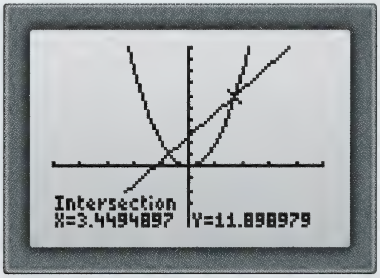


```
Intersection
X=⁻1.44949   Y=2.1010205
```

Step 5: Repeat Steps 1 to 4 to find the other intersection point.



```
Y2=2X+5
Guess?
X=3.4042553  Y=11.808511
```



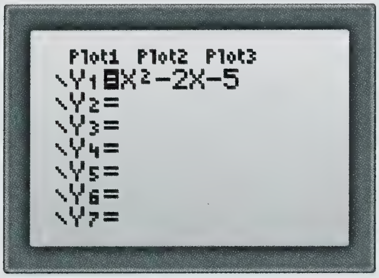
```
Intersection
X=3.4494897  Y=11.898979
```

The two points of intersection are approximately $(-1.45, 2.10)$ and $(3.45, 11.90)$.

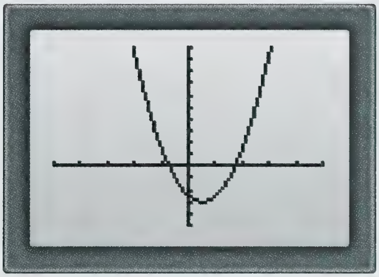
You know there are no more intersection points because the gaph of $y = 2x + 5$ is a straight line and is moving away from the graph of $y = x^2$.

3. **Textbook exercises 4 to 8 of “Investigation 1: Intersecting Graphs,” p. 95**

4.

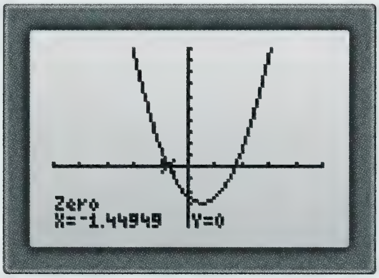


```
Plot1 Plot2 Plot3
\Y1=X²-2X-5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

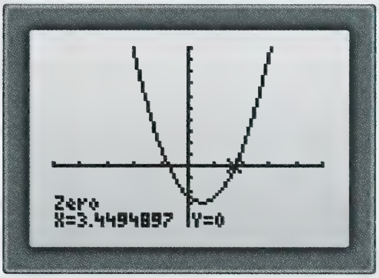


The function $y=x^2-2x-5$ is greater than zero when x^2 is greater than $2x+5$. When it is less than zero, $2x+5$ is greater than x^2.

5. Use the Zero feature on your graphing calculator to find the *x*-intercept.



```
Zero
X=-1.44949   Y=0
```



```
Zero
X=3.4494897  Y=0
```

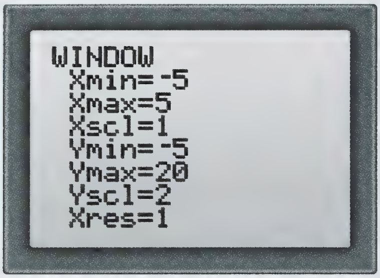
The zeros of the function $y=x^2-2x-5$ are approximately -1.45 and 3.45. The zeros are the *x*-coordinates of the *x*-intercepts of $y=x^2-2x-5$.

6. The intersection points found in exercises 2 and 3, $(-1.45, 2.10)$ and $(3.45, 11.90)$, have the same *x*-coordinates as the zeros of $y=x^2-2x-5$.

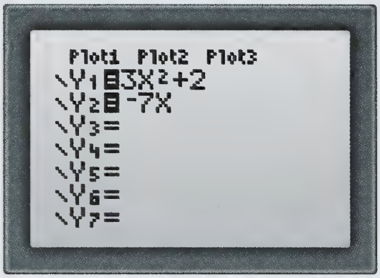
7. Two methods to solve $x^2=5x-4$ are as follows:

- **Method 1:** Finding the intersection points of $y=x^2$ and $y=5x-4$
- **Method 2:** Finding the zeros of $y=x^2-5x+4$

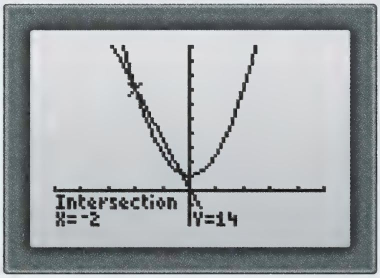
8. a. **Method 1: Finding the Intersection Points**



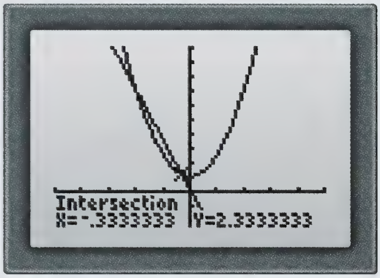
```
WINDOW
 Xmin=-5
 Xmax=5
 Xscl=1
 Ymin=-5
 Ymax=20
 Yscl=2
 Xres=1
```



```
Plot1 Plot2 Plot3
\Y1=3X²+2
\Y2=-7X
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



```
Intersection
X=-2        Y=14
```



```
Intersection
X=-.3333333  Y=2.3333333
```

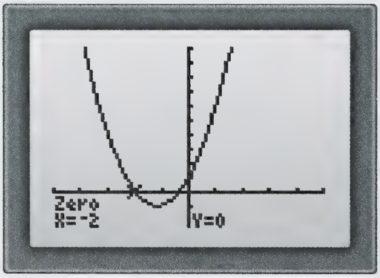
$\therefore\ x=-2$ and $x \doteq -0.33$

Method 2: Finding the Zeros

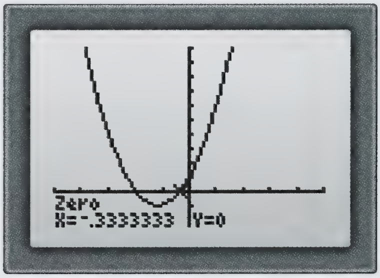
First, bring all of the terms to one side.

$$3x^2+2=-7x$$
$$3x^2+7x+2=0$$

Now, graph the function $y=3x^2+7x+2$, and find the zeros.



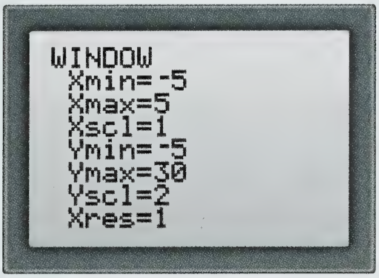
```
Zero
X=-2        Y=0
```



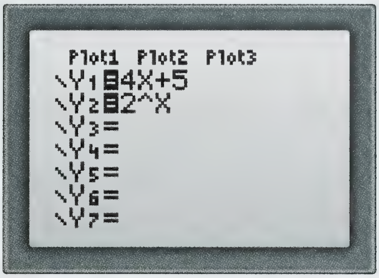
```
Zero
X=-.3333333  Y=0
```

$\therefore\ x=-2$ and $x \doteq -0.33$

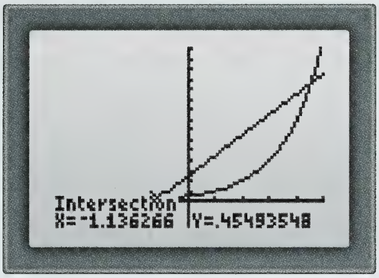
b. Method 1: Finding the Intersection Points



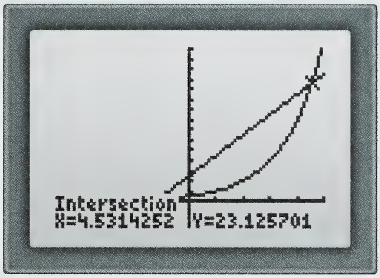
```
WINDOW
 Xmin=-5
 Xmax=5
 Xscl=1
 Ymin=-5
 Ymax=30
 Yscl=2
 Xres=1
```



```
 Plot1 Plot2 Plot3
\Y1=4X+5
\Y2=2^X
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



```
Intersection
X=-1.136266  Y=.45493548
```



```
Intersection
X=4.5314252  Y=23.125701
```

$\therefore\ x = -1.14$ and $x \doteq 4.53$

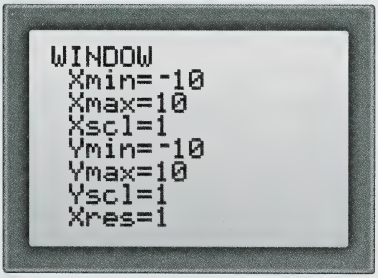
Method 2: Finding the Zeros

First, bring all of the terms to one side.

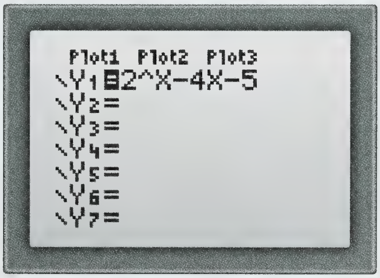
$$4x+5=2^x$$

$$5=2^x-4x$$

$$0=2^x-4x-5$$



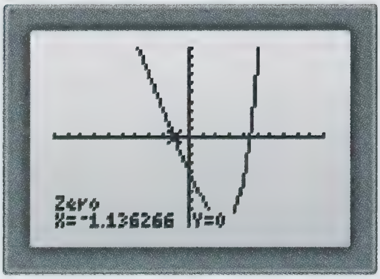
```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```



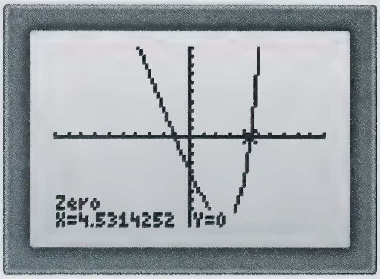
```
 Plot1 Plot2 Plot3
\Y1=2^X-4X-5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Activity 4 (continued)

Now, graph the function $y = 2^x - 4x - 5$ using the given window settings, and find the zeros.



Zero
X=-1.136266 Y=0



Zero
X=4.5314252 Y=0

$\therefore\ x = -1.14$ and $x \doteq 4.53$

4. Textbook exercises 1, 3, and 7 of "Discussing the Ideas," p. 99

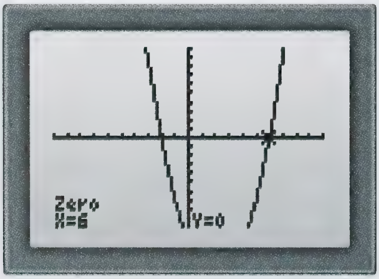
1. The equation $y = x^2$ is non-linear; so, the system, which includes this equation, is also considered to be non-linear.

3. The equation $x^3 + x - 30 = 0$ can be solved by solving either of these systems of equations:

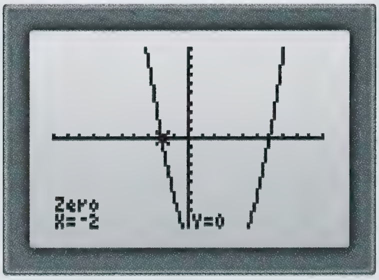
- $y = x^3$
  $y = -x + 30$
- $y = x^3 + x$
  $y = 30$
- $y = x^3 - 30$
  $y = -x$

7. You can tell the roots are exact when the calculator shows the x-values of the zeros or of the intersection points as whole numbers or as repeating decimals.

For example, for the non-linear equation $x^2 - 4x - 12 = 0$, the calculator shows the zeros to be the following.



Zero
X=6 Y=0



Zero
X=-2 Y=0

5. Textbook exercises 1.c., 3.a., and 3.b. of "Exercises: Checking Your Skills," p. 100

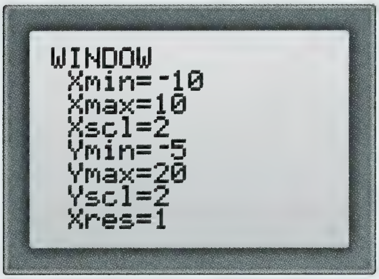
1. c. Enter the following system of equations into your graphing calculator.

$$y = x^3 + x$$
$$y = 15$$

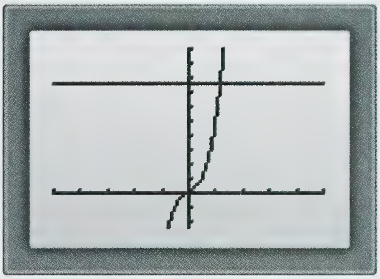
You could also use these systems to solve the problem:

- $y = x^3$
  $y = -x + 15$
- $y = x^3 - 15$
  $y = -x$

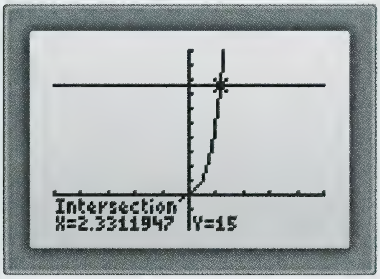
Using the window settings given, the graph of this system looks like the following.



```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=2
 Ymin=-5
 Ymax=20
 Yscl=2
 Xres=1
```



Use the Intersect feature from the CALCULATE menu to find where the two graphs intersect.



```
Intersection
X=2.3311947  Y=15
```

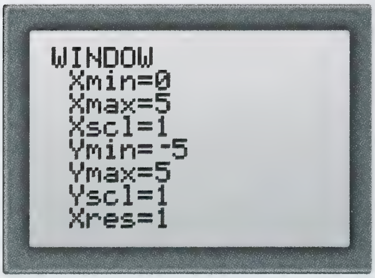
The *x*-value of the intersection is about 2.33. Therefore, the solution to the equation $x^3 + x - 15 = 0$ is $x \doteq 2.33$.

3. a. The value of *h* when Pelletier hits the water is 0. The quadratic equation that represents this situation is $-4.9t^2 + 8.8t + 3 = 0$.

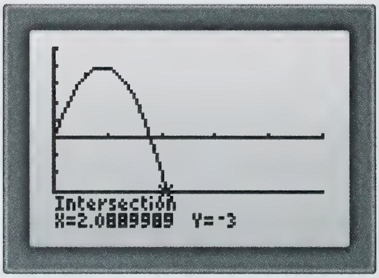
Activity 4 (continued)

b. Method 1: Finding the Point of Intersection

Find the intersection point of $y = -4.9x^2 + 8.8x$ and $y = -3$ to determine the time of flight of Pelletier's dive.



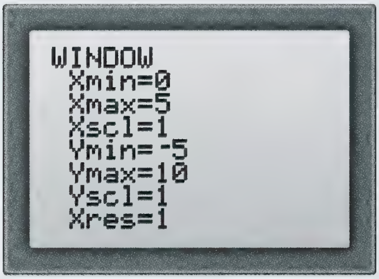
```
WINDOW
 Xmin=0
 Xmax=5
 Xscl=1
 Ymin=-5
 Ymax=5
 Yscl=1
 Xres=1
```



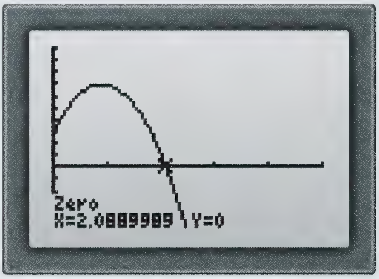
```
Intersection
X=2.0889989  Y=-3
```

Method 2: Finding the Zeros

Graph the function $y = -4.9x^2 + 8.8t + 3$ using the given window settings, and find the zeros.



```
WINDOW
 Xmin=0
 Xmax=5
 Xscl=1
 Ymin=-5
 Ymax=10
 Yscl=1
 Xres=1
```



```
Zero
X=2.0889989  Y=0
```

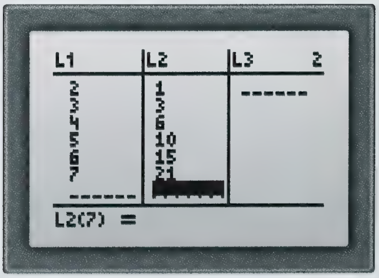
The time of flight of Pelletier's dive was about 2.09 s.

6. **Textbook exercises A, B, and C of "Investigation 2: Modelling Real Situations Using Regression Techniques," pp. 96 and 97**

Part A

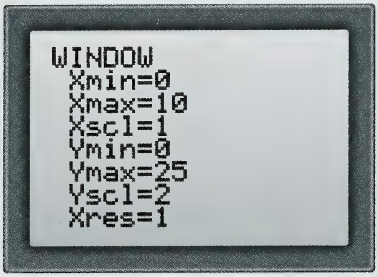
a.

Number of subscribers	2	3	4	5	6	7
Number of links	1	3	6	10	15	21

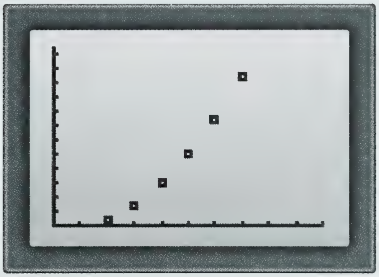


```
L1      L2      L3     2
2       1       ------
3       3
4       6
5       10
6       15
7       21
------
L2(7) =
```

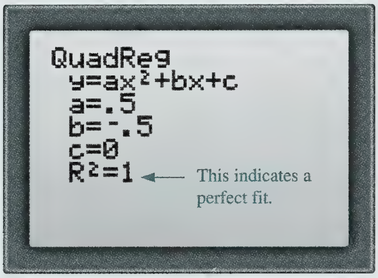
b.



```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=1
 Ymin=0
 Ymax=25
 Yscl=2
 Xres=1
```

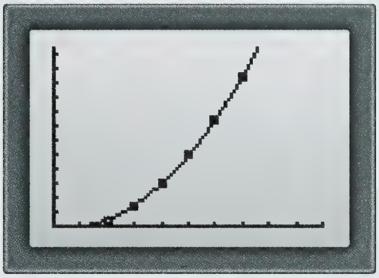


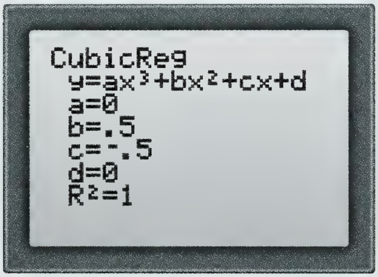
c.



```
QuadReg
 y=ax²+bx+c
 a=.5
 b=-.5
 c=0
 R²=1
```

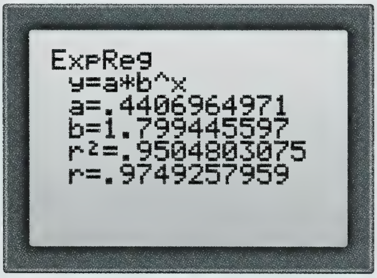
This indicates a perfect fit.



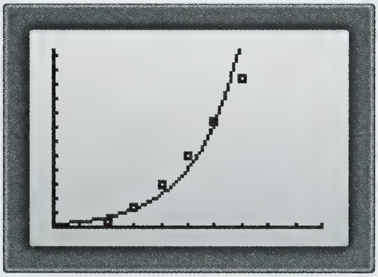


```
CubicReg
 y=ax³+bx²+cx+d
 a=0
 b=.5
 c=-.5
 d=0
 R²=1
```

Because $a = 0$, this gives the same function as QuadReg.

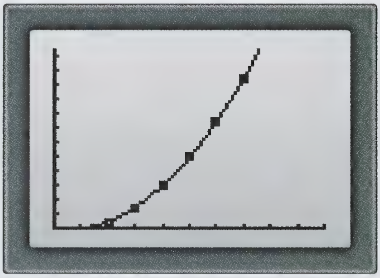


```
ExpReg
 y=a*b^x
 a=.4406964971
 b=1.799445597
 r²=.9504803075
 r=.9749257959
```



Activity 4 (continued)

d. The quadratic regression curve fits perfectly. Therefore, the quadratic function $y = 0.5x^2 - 0.5x$ should be used to model the data.



Part B

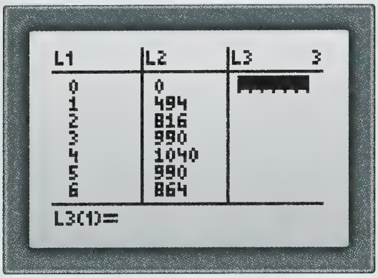
a. When larger squares are removed, the base becomes smaller and the height increases.

b.

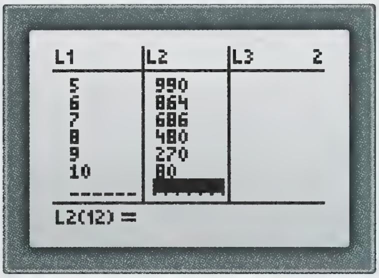
Side Length of Square Removed	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
0	28	21	0	0
1	26	19	1	494
2	24	17	2	816
3	22	15	3	990
4	20	13	4	1040
5	18	11	5	990
6	16	9	6	864
7	14	7	7	686
8	12	5	8	480
9	10	3	9	270
10	8	1	10	80

Recall: $V = l \times w \times h$

With L1 as side length and L2 as volume, the data is entered as follows.

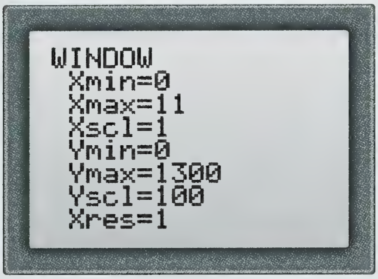


```
L1     L2     L3    3
0      0
1      494
2      816
3      990
4      1040
5      990
6      864
L3(1)=
```

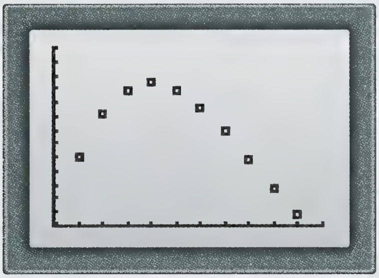


```
L1     L2     L3    2
5      990
6      864
7      686
8      480
9      270
10     80
------
L2(12) =
```

c.

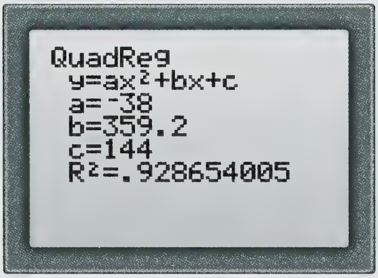


```
WINDOW
 Xmin=0
 Xmax=11
 Xscl=1
 Ymin=0
 Ymax=1300
 Yscl=100
 Xres=1
```

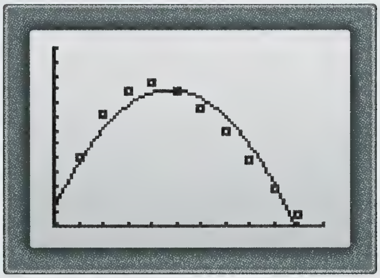


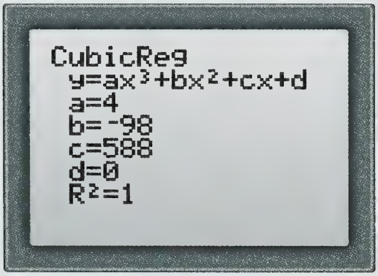
Note: You only need Xmax set to 11 because each piece of cardboard is only 21 cm wide. Therefore, the largest square that can be cut out is one with sides measuring 10.5 cm.

d.

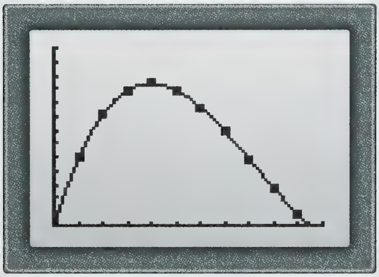


```
QuadReg
 y=ax²+bx+c
 a=-38
 b=359.2
 c=144
 R²=.928654005
```





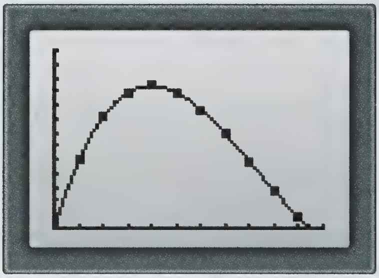
```
CubicReg
 y=ax³+bx²+cx+d
 a=4
 b=-98
 c=588
 d=0
 R²=1
```



The cubic regression curve fits perfectly.

Activity 4 (continued)

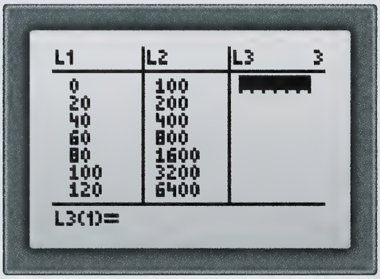
e. The cubic function $y = 4x^3 - 98x^2 + 588x$ should be used to model the data.



Part C

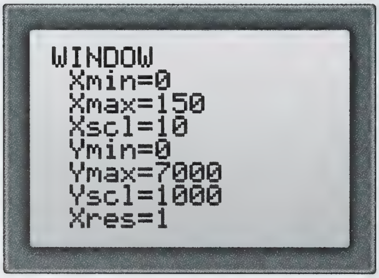
a.

Time (min)	Number of Bacteria	
0	100	
20	200	×2
40	400	×2
60	800	×2
80	1600	×2
100	3200	×2
120	6400	×2

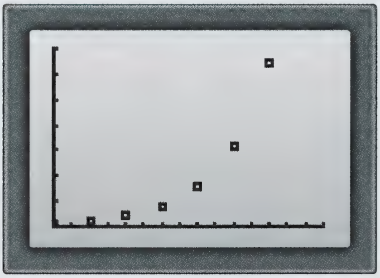


```
L1     L2     L3     3
0      100
20     200
40     400
60     800
80     1600
100    3200
120    6400
L3(1)=
```

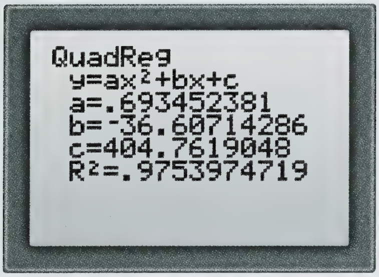
b.



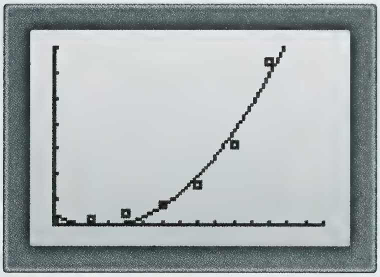
```
WINDOW
 Xmin=0
 Xmax=150
 Xscl=10
 Ymin=0
 Ymax=7000
 Yscl=1000
 Xres=1
```

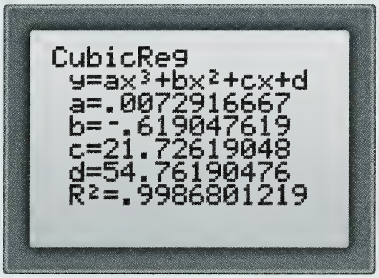


c.

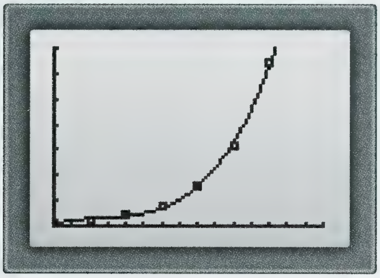


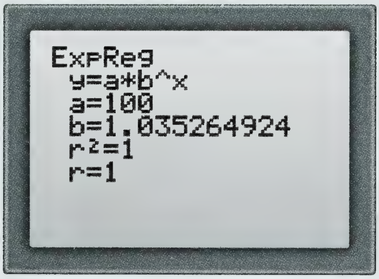
```
QuadReg
 y=ax²+bx+c
 a=.693452381
 b=-36.60714286
 c=404.7619048
 R²=.9753974719
```



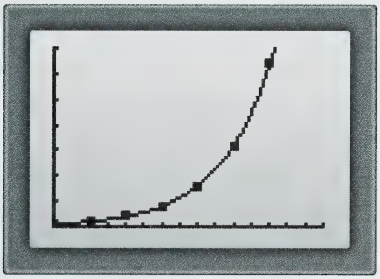


```
CubicReg
 y=ax³+bx²+cx+d
 a=.0072916667
 b=-.619047619
 c=21.72619048
 d=54.76190476
 R²=.9986801219
```



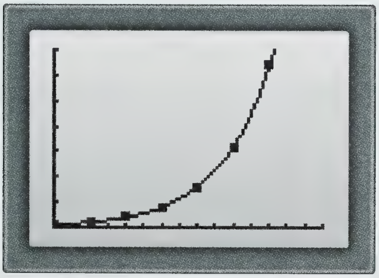


```
ExpReg
 y=a*b^x
 a=100
 b=1.035264924
 r²=1
 r=1
```



The exponential regression curve fits perfectly.

d. The exponential function $y=100(1.035)^x$ should be used to model the data.



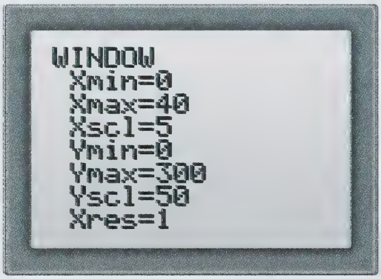
7. Textbook exercises 2 to 4 of "Investigation 2: Modelling Real Situations Using Regression Techniques," pp. 97 and 98

2. The function that best fit the data was $y = 0.5x^2 - 0.5$. Because there is enough material for 200 links, substitute $y = 200$ into the function and determine the largest community that can be serviced.

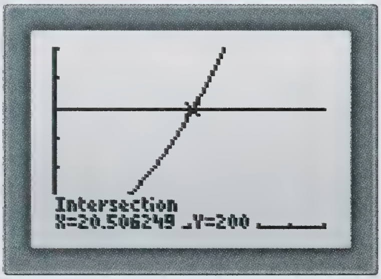
$$y = 0.5x^2 - 0.5x$$
$$200 = 0.5x^2 - 0.5x$$

Method 1: Finding the Intersection Points

Graph $y = 0.5x^2 - 0.5x$ as Y_1 and $y = 200$ as Y_2; then determine where the two graphs intersect using the Intersect feature from the CALCULATE menu.



```
WINDOW
 Xmin=0
 Xmax=40
 Xscl=5
 Ymin=0
 Ymax=300
 Yscl=50
 Xres=1
```

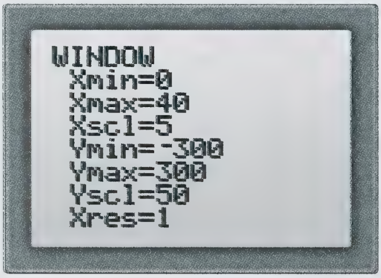


```
Intersection
X=20.506249  Y=200
```

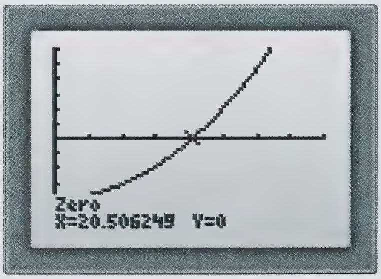
The largest community that can be served by the company is one with 20 subscribers.

Method 2: Finding the *x*-Intercept

Graph $y = 0.5x^2 - 0.5x - 200$; then find the *x*-intercept using the Zero feature from the CALCULATE menu.



```
WINDOW
 Xmin=0
 Xmax=40
 Xscl=5
 Ymin=-300
 Ymax=300
 Yscl=50
 Xres=1
```



```
Zero
X=20.506249  Y=0
```

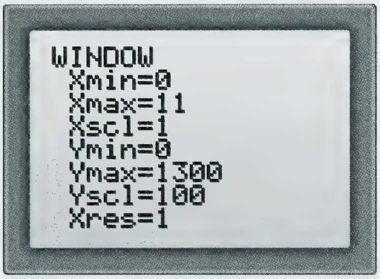
The largest community that can be served by the company is one with 20 subscribers.

3. The function that best fit the data was $y = 4x^3 - 98x^2 + 588x$. Because the desired volume is 750 cm^3, substitute $y = 750$ into the function and determine the dimensions of the squares to be cut out of the corners of the cardboard. (These dimensions correspond to the the height of the box.)

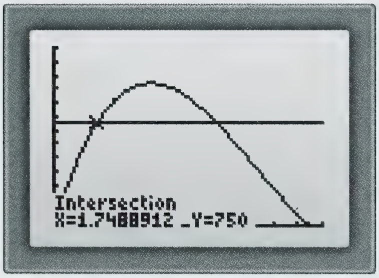
$$\begin{aligned} y &= 4x^3 - 98x^2 + 588x \\ 750 &= 4x^3 - 98x^2 + 588x \end{aligned}$$

Method 1: Finding the Intersection Points

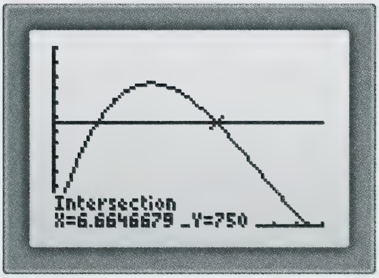
Graph $y = 4x^3 - 98x^2 + 588x$ as Y_1 and $y = 750$ as Y_2; then determine where the two graphs intersect using the Intersect feature from the CALCULATE menu.



```
WINDOW
 Xmin=0
 Xmax=11
 Xscl=1
 Ymin=0
 Ymax=1300
 Yscl=100
 Xres=1
```



```
Intersection
X=1.7488912  Y=750
```

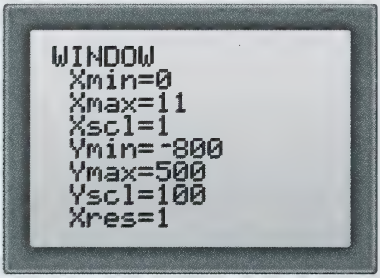


```
Intersection
X=6.6646679  Y=750
```

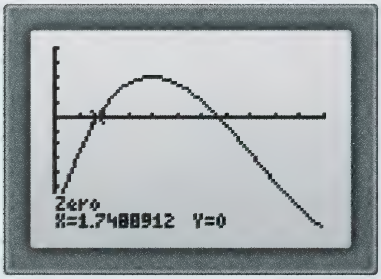
A 750-cm^3 box can be obtained by removing squares with sides measuring approximately 1.75 cm or 6.66 cm.

Method 2: Finding the x-Intercept

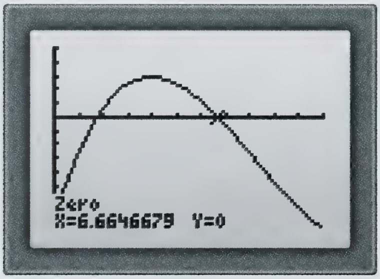
Graph $y = 4x^3 - 98x^2 + 588x - 750$; then find the x-intercept using the Zero feature from the CALCULATE menu.



```
WINDOW
 Xmin=0
 Xmax=11
 Xscl=1
 Ymin=-800
 Ymax=500
 Yscl=100
 Xres=1
```



```
Zero
X=1.7488912  Y=0
```



```
Zero
X=6.6646679  Y=0
```

A 750-cm^3 box can be obtained by removing squares with sides measuring approximately 1.75 cm or 6.66 cm.

Now, determine the dimensions of each box.

When $x \doteq 1.75$, Dimensions $= \ell \times w \times h$

$$
\begin{aligned}
&\doteq [28 - 2(1.75)] \times [21 - 2(1.75)] \times 1.75 \\
&\doteq 24.50 \times 17.50 \times 1.75
\end{aligned}
$$

When $x \doteq 6.66$, Dimensions $= \ell \times w \times h$

$$
\begin{aligned}
&\doteq [28 - 2(6.66)] \times [21 - 2(6.66)] \times 6.66 \\
&\doteq 14.68 \times 7.68 \times 6.66
\end{aligned}
$$

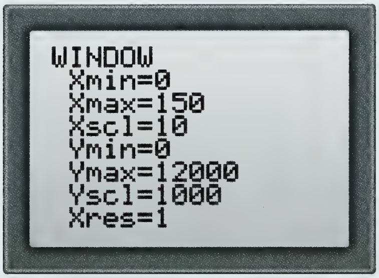
The dimensions of the boxes that yield a volume of $750\ \text{cm}^3$ are approximately $24.50\ \text{cm} \times 17.50\ \text{cm} \times 1.75\ \text{cm}$ and $14.68\ \text{cm} \times 7.68\ \text{cm} \times 6.66\ \text{cm}$.

4. The function that best fit the data was $y = 100(1.035\,264\,924)^x$. Because the researcher needs to know when the number of bacteria reaches 10 000, substitute $y = 10\,000$ into the function and determine the time (in minutes).

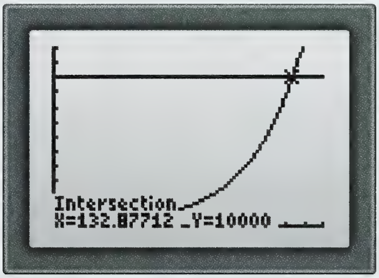
$$y = 100(1.035\,264\,924)^x$$
$$10\,000 = 100(1.035\,264\,924)^x$$

Method 1: Finding the Intersection Points

Graph $y = 100(1.035\,264\,924)^x$ as Y_1 and $y = 10\,000$ as Y_2; then determine where the two graphs intersect using the Intersect feature from the CALCULATE menu.



```
WINDOW
 Xmin=0
 Xmax=150
 Xscl=10
 Ymin=0
 Ymax=12000
 Yscl=1000
 Xres=1
```

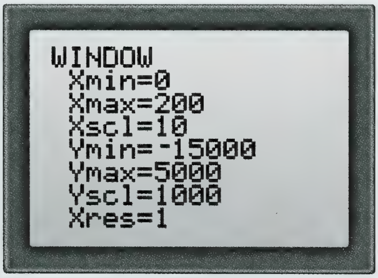


```
Intersection
X=132.87712  Y=10000
```

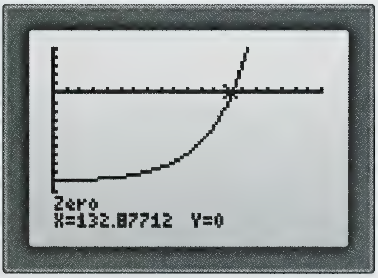
The number of bacteria reaches 10 000 after approximately 133 min.

Method 2: Finding the *x*-Intercept

Graph $y = 100(1.035\,264\,924)^x - 10\,000$; then find the *x*-intercept using the Zero feature from the CALCULATE menu.



```
WINDOW
 Xmin=0
 Xmax=200
 Xscl=10
 Ymin=-15000
 Ymax=5000
 Yscl=1000
 Xres=1
```



```
Zero
X=132.87712  Y=0
```

The number of bacteria reaches 10 000 after approximately 133 min.

Activity 4 (continued)

8. Textbook exercises 4, 5, and 10 of “Exercises: Checking Your Skills,” pp. 100 to 102

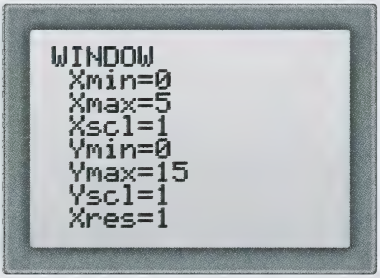
4. **a.** The outside diameter of the washer is shown to be 4 cm. Therefore, the outside radius is $4 \div 2 = 2$ cm.

$$\begin{aligned} \therefore\ A &= \pi R^2 - \pi r^2 \\ &= \pi 2^2 - \pi r^2 \quad \longleftarrow \text{Substitute 2 for } R. \\ &= 4\pi - \pi r^2 \end{aligned}$$

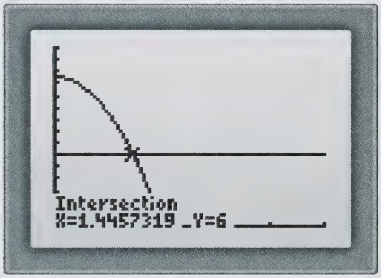
b. The equation $4\pi - \pi r^2 = 6$ represents the situation here.

This equation can be solved by solving this system of equations:

$$y = 4\pi - \pi x^2$$
$$y = 6$$



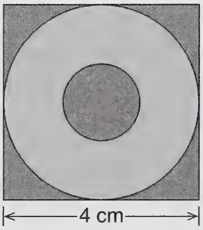
```
WINDOW
 Xmin=0
 Xmax=5
 Xscl=1
 Ymin=0
 Ymax=15
 Yscl=1
 Xres=1
```



```
Intersection
X=1.4457319  Y=6
```

The washer has an area of 6 cm^2 when the inside radius is about 1.45 cm.

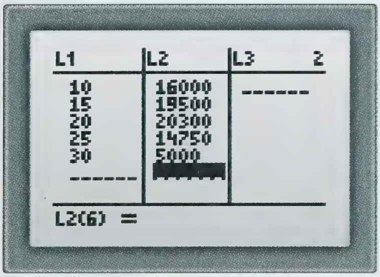
c. The area of the metal wasted is equal to the total area of the square minus the area of the washer.



$$\begin{aligned} \text{Wasted area} &= \text{Area of square} - \text{Area of washer} \\ &= 16 - 6 \\ &= 10 \end{aligned}$$

The area of metal wasted is 10 cm^2.

5. **a.** **Step 1:** Enter the data as L1 and L2.

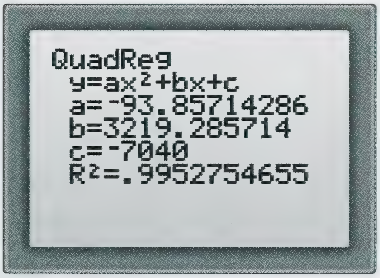


```
L1      |L2      |L3      2
 10     | 16000  | ------
 15     | 19500  |
 20     | 20300  |
 25     | 14750  |
 30     | 5000   |
 ------ |        |
L2(6) =
```

Step 2: Determine the best-fit quadratic function for the data.

STAT ▶ 5 (5:QuadReg)

Select the CALC menu.

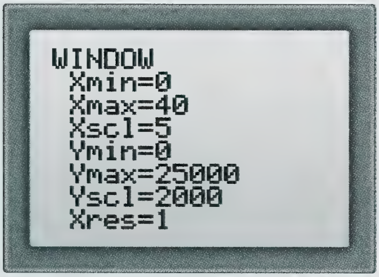


```
QuadReg
 y=ax²+bx+c
 a=⁻93.85714286
 b=3219.285714
 c=⁻7040
 R²=.9952754655
```

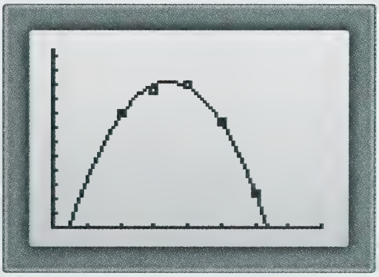
Step 3: Using the given window settings, visually confirm the equation by graphing the data and graph together.

Select the EQ menu.

Y= VARS 5 (5:Statistics…) ▶ ▶ 1 (1:RegEQ) GRAPH



```
WINDOW
 Xmin=0
 Xmax=40
 Xscl=5
 Ymin=0
 Ymax=25000
 Yscl=2000
 Xres=1
```



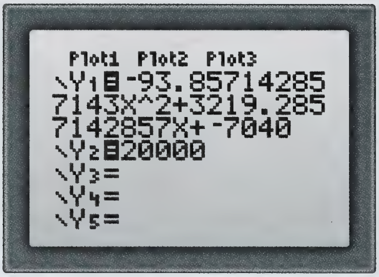
The quadratic function that best fits the data is $y = -93.86x^2 + 3219.29x - 7040$.

Activity 4 (continued)

b. Solve the following system of equations:

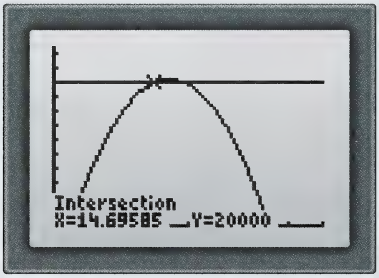
$$y = -93.86x^2 + 3219.29 - 7040$$

$$y = 20\,000$$

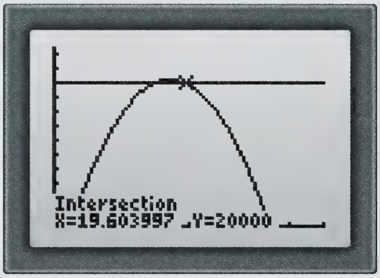


```
Plot1 Plot2 Plot3
\Y1■-93.85714285
7143X^2+3219.285
7142857X+-7040
\Y2■20000
\Y3=
\Y4=
\Y5=
```

The solution corresponds to the *x*-values of the intersection points. Use the Intersect feature from the CALCULATE menu.



```
Intersection
X=14.69585   Y=20000
```

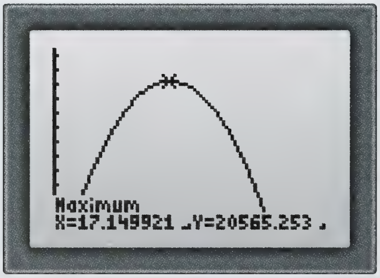


```
Intersection
X=19.603997  Y=20000
```

To generate a revenue of $20 000, the ticket price should be $14.70 or $19.60.

c. The revenue will be greater between ticket prices of $14.70 and $19.60.

d. Use the Maximum feature from the CALCULATE menu.



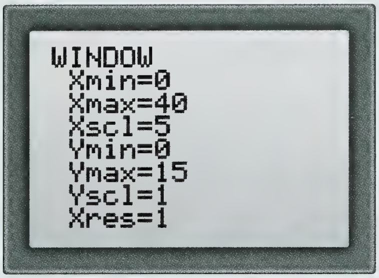
```
Maximum
X=17.149921  Y=20565.253
```

The optimum ticket price is $17.15. This price will generate the maximum revenue.

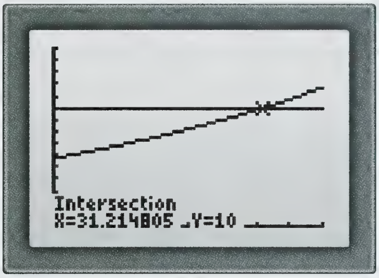
10. To predict the year when the world's population will reach 10 billion, solve this system of equations:

$$y = 5.73(1.018)^x$$
$$y = 10$$

The x-value of the intersection point is the solution of the system. Use the Intersect feature from the CALCULATE menu.



```
WINDOW
 Xmin=0
 Xmax=40
 Xscl=5
 Ymin=0
 Ymax=15
 Yscl=1
 Xres=1
```

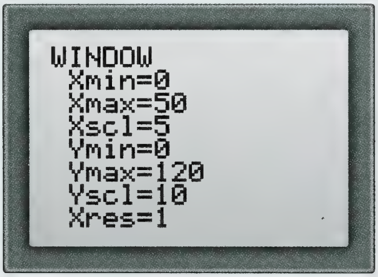


```
Intersection
X=31.214805  Y=10
```

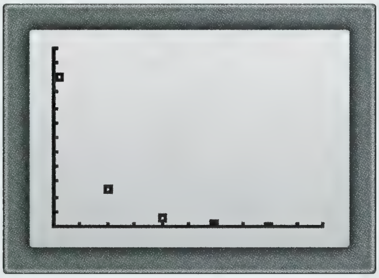
The world's population will reach 10 billion about 31 years after 1995, or in 2026.

9. Textbook exercise 6 of Part B of "What Should I Be Able To Do?," p. 108

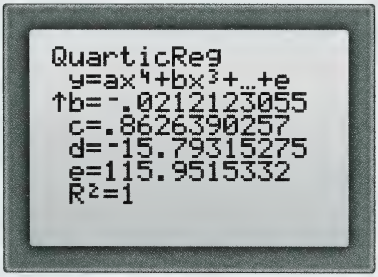
6. a.



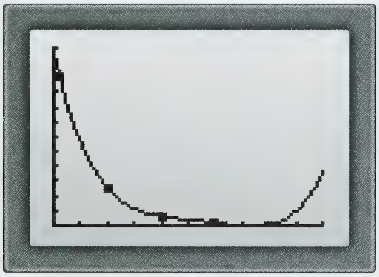
```
WINDOW
 Xmin=0
 Xmax=50
 Xscl=5
 Ymin=0
 Ymax=120
 Yscl=10
 Xres=1
```



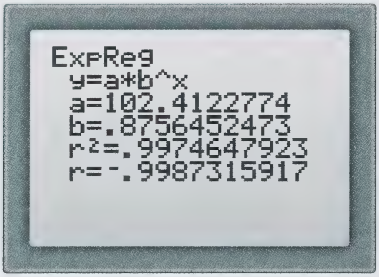
b.



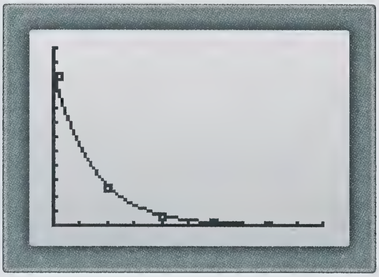
```
QuarticReg
 y=ax⁴+bx³+…+e
↑b=⁻.0212123055
 c=.8626390257
 d=⁻15.79315275
 e=115.9515332
 R²=1
```



A quartic function fits the data "perfectly." However, you would expect the atmospheric pressure to decrease as the altitude increases. The graph shows that this function does not continue along the expected trend; it shows the pressure increasing with an increase in altitude beyond the last data point. So, try another function type.



```
ExpReg
 y=a*b^x
 a=102.4122774
 b=.8756452473
 r²=.9974647923
 r=-.9987315917
```

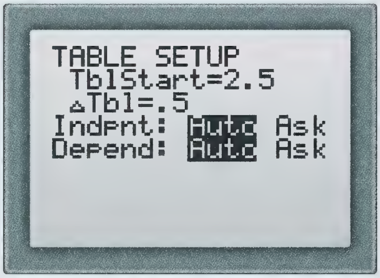


The Exponential Regression feature provides a very close fit function that follows the expected trend beyond the data. The fact that it does show pressure decreasing as altitude increases and that it is the best fit for the data (other than QuartReg) makes this the best-fit function.

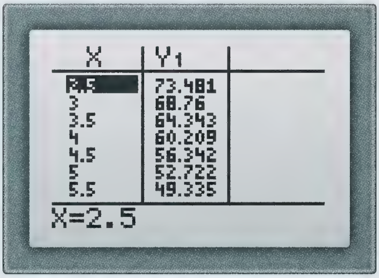
Therefore, the function that best represents the data is $y = 102.41(0.88)^x$.

c. **Method 1: Using the Table of Values**

Adjust the TABLE SETUP window as indicated, and display the table of values.



```
TABLE SETUP
 TblStart=2.5
 ΔTbl=.5
Indpnt: Auto Ask
Depend: Auto Ask
```

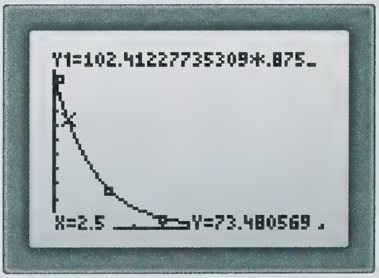


```
  X   | Y1
2.5   | 73.481
3     | 68.76
3.5   | 64.343
4     | 60.209
4.5   | 56.342
5     | 52.722
5.5   | 49.335
X=2.5
```

The air pressure at a height of 2.5 km is approximately 73.5 kPa.

Method 2: Using the Value Feature

Select the Value feature from the CALCULATE menu by pressing 2nd [CALC] 1 (1:value). When the graph appears and the calculator prompts you to enter a value, enter 2.5 for x and press ENTER.



```
Y1=102.41227735309*.875_
X=2.5          Y=73.480569
```

The air pressure at a height of 2.5 km is approximately 73.5 kPa.

10. Textbook exercise “Communicating the Ideas,” p. 103

The following are two methods of solving a non-linear equation using a graphing calculator:

Method 1: Using Intersecting Graphs

Set Y_1 equal to the left side of the original equation and Y_2 equal to the right side. Then find the intersection points of the graphs corresponding to Y_1 and Y_2. The *x*-values of these intersection points are the solutions to the original equations.

Method 2: Using *x*-Intercepts

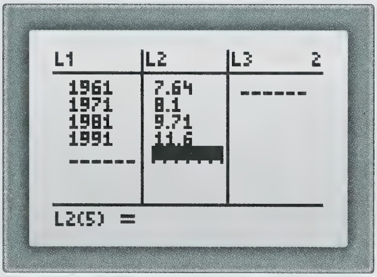
Rearrange the equation so 0 is on one side. Then, in the equation editor, set Y_1 equal to the non-zero side of the rearranged equation. Find the zeros of the equation corresponding to Y_1. These zeros correspond to the solution of the original equation.

Using *x*-intercepts is easier when using equations generated by regression analysis, since the calculator will place the equation into the equation editor for you.

Follow-up Activities

Textbook exercise 1 of Part A of “What Should I Be Able To Do?,” p. 107

1. a.

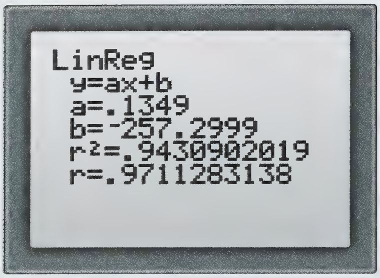


```
L1      |L2      |L3     2
 1961   | 7.64   | ------
 1971   | 8.1    |
 1981   | 9.71   |
 1991   | 11.6   |
 ------ |        |
L2(5) =
```

Follow-up Activities (continued)

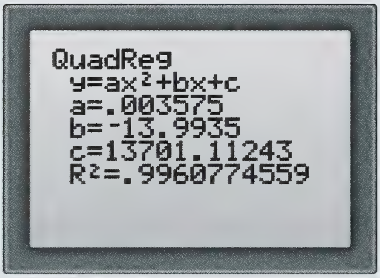
b. Press STAT ▶ (selecting the CALC menu) and choose the regression.

Linear



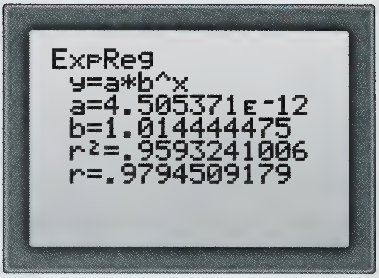
```
LinReg
 y=ax+b
 a=.1349
 b=-257.2999
 r²=.9430902019
 r=.9711283138
```

Quadratic



```
QuadReg
 y=ax²+bx+c
 a=.003575
 b=-13.9935
 c=13701.11243
 R²=.9960774559
```

Exponential

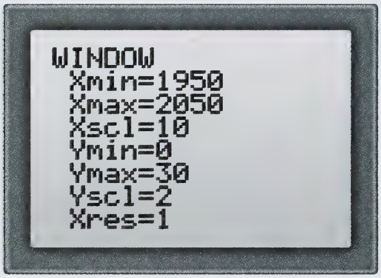


```
ExpReg
 y=a*b^x
 a=4.505371E-12
 b=1.014444475
 r²=.9593241006
 r=.9794509179
```

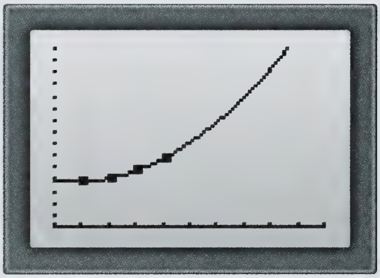
Based on the correlation coefficient, the quadratic function $y = 0.003\,575x^2 - 13.9935x + 13\,701.112\,43$ fits the data best.

c. Y= VARS 5 (5:Statistics…) ▶ ▶ 1 (1:RegEQ) GRAPH

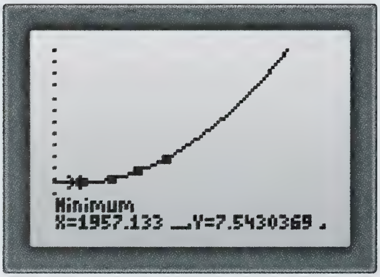
Select the EQ menu.



```
WINDOW
 Xmin=1950
 Xmax=2050
 Xscl=10
 Ymin=0
 Ymax=30
 Yscl=2
 Xres=1
```



d. The quadratic function graph opens upward; so, it has a minimum, not a maximum.



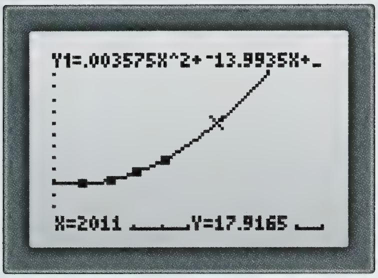
```
Minimum
X=1957.133    Y=7.5430369
```

The graph has a minimum at $(1957, 7.54)$.

e. Method 1: Using the Value Feature

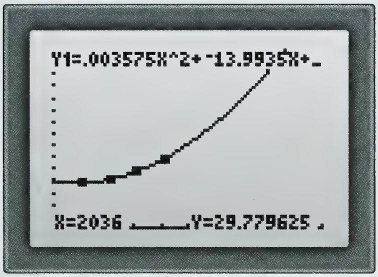
Select the Value feature from the CALCULATE menu, and determine the percentages when $x = 2011$ and when $x = 2036$.

2nd [CALC] 1 (1:value) 2 0 1 1 ENTER



```
Y1=.003575X^2+⁻13.9935X+_
X=2011        Y=17.9165
```

2 0 3 6 ENTER



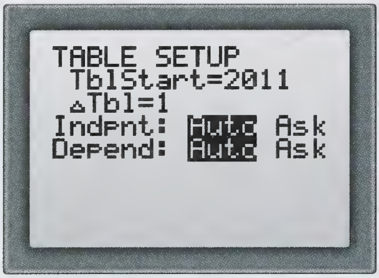
```
Y1=.003575X^2+⁻13.9935X+_
X=2036        Y=29.779625
```

Note: Until you are finished using the Value feature, you do not need to press 2nd [CALC] 1 again. You only need to enter the desired *x*-value.

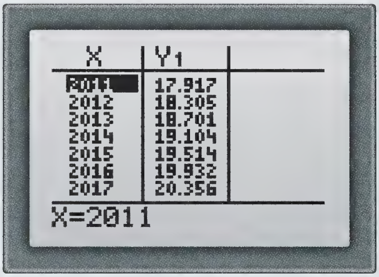
This equation predicts that the percent of the Canadian population over 65 will be approximately 17.9% in 2011 and approximately 29.8% in 2036.

Method 2: Using the Table of Values

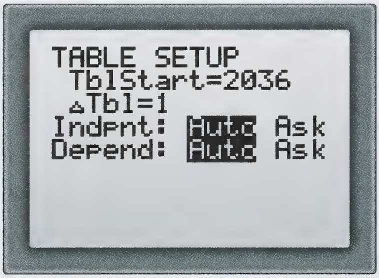
Set up the table of values so it starts at $x = 2011$; then read the corresponding y-value. Do the same for $x = 2036$.



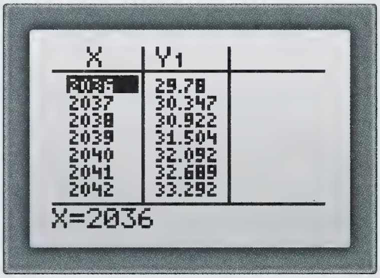
```
TABLE SETUP
 TblStart=2011
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```



```
   X     Y1
 2011   17.917
 2012   18.305
 2013   18.701
 2014   19.104
 2015   19.514
 2016   19.932
 2017   20.356
X=2011
```



```
TABLE SETUP
 TblStart=2036
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

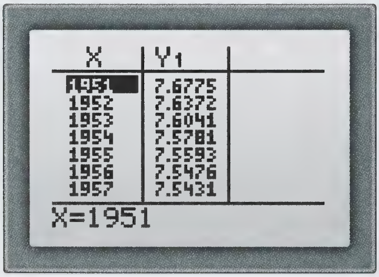


```
   X     Y1
 2036   29.78
 2037   30.347
 2038   30.922
 2039   31.504
 2040   32.092
 2041   32.689
 2042   33.292
X=2036
```

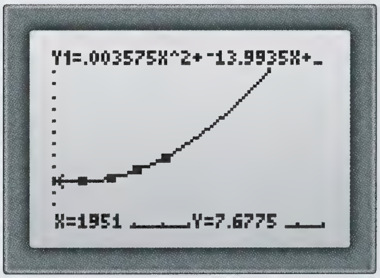
Note: After finding the percent when $x = 2011$, you could have also used the down arrow key to scroll down to $x = 2036$.

This equation predicts that the percent of the Canadian population over 65 will be approximately 17.9% in 2011 and approximately 29.8% in 2036.

f. Use the table of values or the Value feature.



```
   X     Y1
 1951   7.6775
 1952   7.6372
 1953   7.6041
 1954   7.5781
 1955   7.5593
 1956   7.5476
 1957   7.5431
X=1951
```



```
Y1=.003575X^2+⁻13.9935X+…
X=1951          Y=7.6775
```

This equation predicts that in 1951, approximately 7.7% of the Canadian population were over 65.

Extra Help

1. **a.**

$$9x^3 - 5x^4 = 20x^3 + 10$$

$$9x^3 - 5x^4 - 9x^3 = 20x^3 + 10 - 9x^3 \quad \longleftarrow \text{Subtract } 9x^3 \text{ from both sides.}$$

$$-5x^4 = 11x^3 + 10$$

$$-5x^4 + 5x^4 = 11x^3 + 10 + 5x^4 \quad \longleftarrow \text{Add } 5x^4 \text{ to both sides.}$$

$$0 = 5x^4 + 11x^3 + 10$$

The zeros of $f(x) = 5x^4 + 11x^3 + 10$ are the solutions of the original equation.

b.

$$4x^3 - 66 = 20x^2 + 200x$$

$$4x^3 - 66 - 20x^2 = 20x^2 + 200x - 20x^2 \quad \longleftarrow \text{Subtract } 20x^2 \text{ from both sides.}$$

$$4x^3 - 20x^2 - 66 = 200x$$

$$4x^3 - 20x^2 - 66 - 200x = 200x - 200x \quad \longleftarrow \text{Subtract } 200x \text{ from both sides.}$$

$$4x^3 - 20x^2 - 200x - 66 = 0$$

The zeros of $f(x) = 4x^3 - 20x^2 - 200x - 66$ are the solutions of the original equation.

2. Simplify the equation.

$$2x^3 + x^2 + 7 = 8x^2 + 2x$$

$$2x^3 + x^2 + 7 - 8x^2 = 8x^2 + 2x - 8x^2$$

$$2x^3 - 7x^2 + 7 = 2x$$

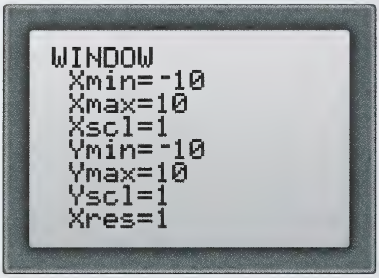
$$2x^3 - 7x^2 + 7 - 2x = 2x - 2x$$

$$2x^3 - 7x^2 - 2x + 7 = 0$$

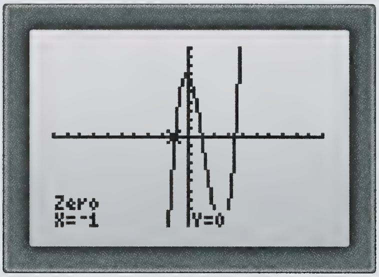
The zeros of $f(x) = 2x^3 - 7x^2 - 2x + 7$ are the solutions to the original equation.

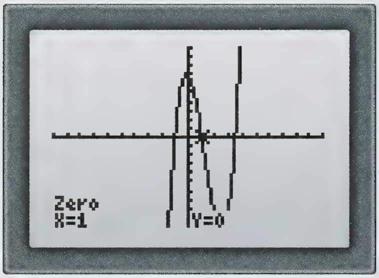
Follow-up Activities (continued)

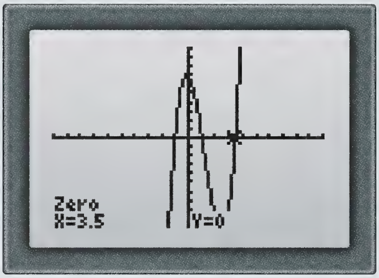
Using the window settings given, graph the function on your graphing calculator; then use the Zero feature from the CALCULATE menu, by pressing 2nd [CALC] 2, to find the zeros.



```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```





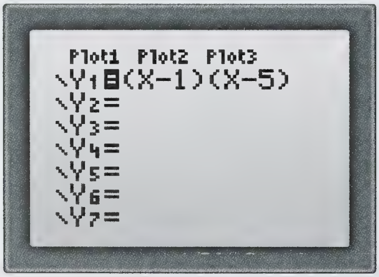


The solutions to the original equation are $x=-1$, $x=1$, and $x=3.5$.

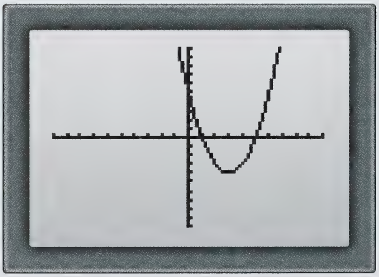
Enrichment

1. **Textbook exercise 7 of "Exercises: Extending Your Thinking," p. 81**

   7. Using the standard window settings, ZOOM 6, the graph of $y=(x-1)(x-5)$ looks like the following.

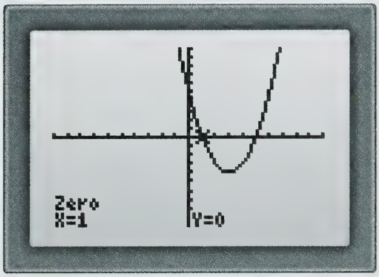


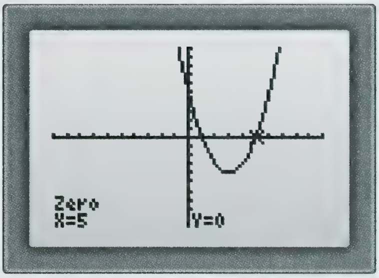
```
Plot1 Plot2 Plot3
\Y1=(X-1)(X-5)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



   a. The function is a quadratic function.

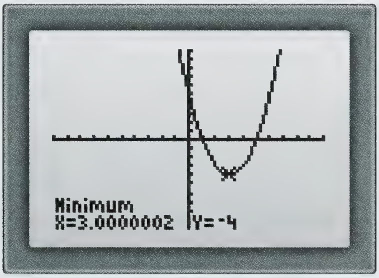
b. Use the Zero feature from the CALCULATE menu.


Zero
X=1
Y=0


Zero
X=5
Y=0

The zeros of the function are $x=1$ and $x=5$.

c. The axis of symmetry goes through the vertex of the graph. Because the vertex of this graph is a minimum, use the Minimum feature from the CALCULATE menu to determine the equation of the axis of symmetry.


Minimum
X=3.0000002
Y=-4

Because the minimum occurs at $(3,-4)$, the equation of the axis of symmetry is $x=3$. Notice that the equation of the axis of symmetry occurs halfway between the zeros $x=1$ and $x=5$.

d. The zeros of a function in factored form $y=(x-k_1)(x-k_2)$ are $x=k_1$ and $x=k_2$.

i. $y=(x-3)(x-7)$

$k_1=3$ ↑ ↑ $k_2=7$

The zeros are $x=3$ and $x=7$. The axis of symmetry is $x=5$.

ii. $y=(x+2)(x-6)$

$=[x-(-2)][x-6]$ ← Convert.

$k_1=-2$ ↑ ↑ $k_2=6$

The zeros are $x=-2$ and $x=6$. The axis of symmetry is $x=2$ (halfway between the zeros).

Follow-up Activities (continued)

2. $y = x^2 + 3x + 2$

$y = (x+1)(x+2)$ ← Factor using trial and error.

$y = \left[x - (-1)\right]\left[x - (-2)\right]$ ← Convert.

$k_1 = -1$ $k_2 = -2$

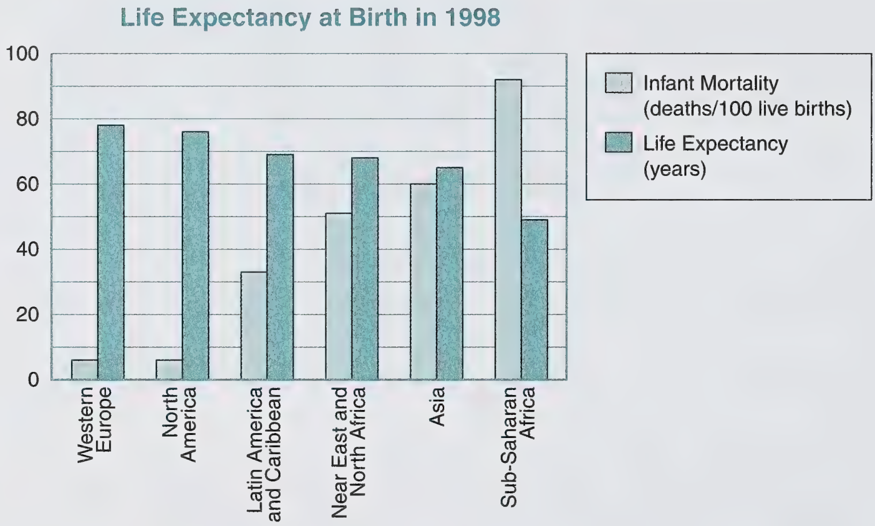
The zeros of $y = x^2 + 3x + 2$ are $x = -1$ and $x = -2$. The axis of symmetry is $x = -\frac{3}{2}$ or -1.5 (halfway between the zeros).

Module Project: The World's Population

Completing the Project

1. **Textbook exercise 1 of "Modelling the Growth of the World's Population," p. 82**

1.



A multiple-bar graph best represents the data because there is a comparison of two groups for each region.

2. **Textbook exercises 8 and 9 of Part C of "What Should I Be Able To Do?," pp. 109 and 110**

8.
- The scales are correct for each axis.
- According to the graph shapes, the correct formula appears to have been used.
- The graphs are clear and easy to read
- The predictions for the years 2000 and 2010 are in exercise 1 of the student sample. Set up the table of values so that TblStart $=10$ and $\Delta\text{Tbl}=10$; then press 2nd [TABLE] and read off the values for 10 and 20. These values correspond to the predictions for the years 2000 and 2010. You could also use the Value feature with $x=10$ and $x=20$. Press 2nd [CALC], enter 10 for x, and press ENTER. Read the predicted value of y.

  The predictions for the years 2000, 2010, and 2020 are also in exercise 2 of the student sample. Set up the table of values so that TblStart $=40$ and $\Delta\text{Tbl}=10$; then read the values for $x=40$, $x=50$, and $x=60$; or use the Value feature with $x=40$, $x=50$, and $x=60$.

  Note: The Trace feature can also be used to check these predictions. You will find that the values for the predictions can be obtained using the Trace feature without the logistic equation.

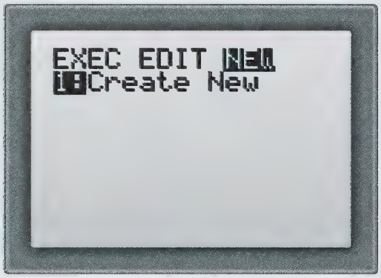
- The values chosen for M are reasonable. The U.N. population division expects the world's population to be 11 billion in 2200. The values chosen are conceivable.

9. This excerpt from the students report is of high quality. The report uses clear statements and presents a logical flow of ideas. After starting from a model of unbounded population growth and the negative consequences of such growth, it is natural to move onto a hopeful scenario involving a stable, sustainable population. The inclusion of graphs to represent population growth helps to convey the mathematical bases for the models.

WORLDPOP

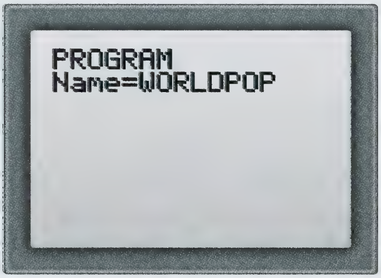
The following shows the keystrokes involved in entering the WORLDPOP program into your TI-83 graphing calculator.

Begin by opening the program window by pressing PRGM. Using the arrow keys, select the NEW menu.



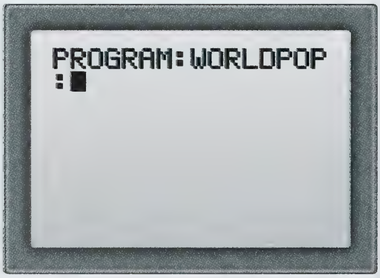
```
EXEC EDIT NEW
1:Create New
```

Press ENTER. Now, the calculator prompts you to name the program. Name the program “WORLDPOP.”



```
PROGRAM
Name=WORLDPOP
```

Press ENTER. Now, the calculator prompts you to enter the lines of the program.



```
PROGRAM:WORLDPOP
:█
```

Enter the lines of the program (as given on page 422 of the textbook) by pressing the following:

2nd [MEM] 4 (4:ClrAllLists) ENTER

0 STO➡ VARS 1 (1:Window…) 1 (1:Xmin) ENTER

1 8 8 STO➡ VARS 1 (1:Window…) 2 (2:Xmax) ENTER

5 0 STO➡ VARS 1 (1:Window…) 3 (3:Xscl) ENTER

0 STO➡ VARS 1 (1:Window…) 4 (4:Ymin) ENTER

2 5 STO➡ VARS 1 (1:Window…) 5 (5:Ymax) ENTER

5 STO➡ VARS 1 (1:Window…) 6 (6:Yscl) ENTER

Select the I/O menu.

PRGM ▶ 3 (3:Disp) 2nd [A-LOCK] [“] [I] [N] [I] [T] [I] [A] [L] [␣] [P] [O] [P] [”]
ENTER

Select the I/O menu.

PRGM ▶ 1 (1:Input) ALPHA [P] ENTER

Select the I/O menu.

PRGM ▶ 3 (3:Disp) 2nd [A-LOCK] [“] [M] [A] [X] [␣] [P] [O] [P] [”] ENTER

Select the I/O menu.

PRGM ▶ 1 (1:Input) ALPHA [M] ENTER

Select the I/O menu.

PRGM ▶ 3 (3:Disp) 2nd [A-LOCK] [“] [R] [A] [T] [E] [”] ENTER

Select the I/O menu.

PRGM ▶ 1 (1:Input) ALPHA [R] ENTER

(ALPHA [R] ÷ 1 0 0) ÷ (ALPHA [M] − ALPHA [P]) STO➡ ALPHA
[A] ENTER

0 STO➡ 2nd [L1] (1) ENTER

ALPHA [P] STO➡ 2nd [L2] (1) ENTER

WORLDPOP (continued)

PRGM 4 (4:For() ALPHA [N] , 1 , 4 0) ENTER

PRGM 4 (4:For() ALPHA [I] , 1 , 1 0) ENTER

ALPHA [P] + ALPHA [A] × ALPHA [P] × (ALPHA [M] − ALPHA [P]) STO➔

ALPHA [P] ENTER

PRGM 7 (7:End) ENTER

1 0 × ALPHA [N] STO➔ 2nd [L1] (1 + ALPHA [N]) ENTER

ALPHA [P] STO➔ 2nd [L2] (1 + ALPHA [N]) ENTER

PRGM 7 (7:End) 2nd [QUIT]

To run the program, press PRGM and select "WORLDPOP" under the EXEC menu.

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